THE EARTH'S VARIABLE ROTATION: SOME GEOPHYSICAL CAUSES

KURT LAMBECK
Australian National University
Canberra, Australia

1. INTRODUCTION

The Earth's variable rotation, its departures from what it would be if it were a rigid body rotating in isolation, has occupied the interest of astronomers and geophysicists for more than 100 years. The reason for this is quite clear when one becomes aware of the range of processes that perturb the Earth from uniform rotation (Figure 1). A complete understanding of the driving mechanisms requires a study of the deformation of the solid Earth, of fluid motions in the core and of the magnetic field, of the mass redistributions and motions within the oceans and atmosphere, and of the interactions between the solid and fluid regions of the planet. The discussion of evidence for the variable rotation includes the examination of not only a variety of optical telescope evidence that goes back some 300 years, but also of historical records of lunar and solar eclipses, and planetary occultations and conjunctions for perhaps the past three millenia. The geological record, in the form of fossil growth rhythms in organisms such as corals, bivalves or brachiopods or as cyclic organic growth and sediment sequences such as stromatolites or banded iron formations, extend, albeit with very considerable uncertainty, the record back through Phanerozoic time and into the Precambrian. To this variety of measurement techniques now has to be added the new methods derived from the space-oriented technological developments of the past few decades. One of the first geophysical conclusions drawn from the Earth's rotation was by W. Hopkins in 1839 who argued that the then widely held view by geologists, that the Earth was essentially fluid below the crust, was inconsistent with the astronomical observations of the precession constant. Observations of the Earth's irregular rotation also provide global constraints on the structure of the Earth; a consequence of the Earth being neither rigid nor an isolated system, of deformations and motions within the body, and of torques acting on it. Examples of mass redistribution include the rotational and tidal deformations of the planet, the seasonal and long period exchange of water mass between the oceans, atmosphere, ice sheets and ground water, and the large scale redistribution of mass associated with the stress cycle of large earthquakes. Examples of torques include the gravitational tidal torques, electromagnetic torques acting on the base of the mantle, and winds and ocean current acting on the surface.

Figure 1 illustrates schematically the variety of phenomena that perturb the Earth's rotation. The geophysical mechanisms may exhibit a variety of wavelengths but the Earth's response is primarily controlled by the global properties of the planet, properties that can often be measured.
more directly and with greater precision by other geophysical techniques. Precise observations of the rotation nevertheless provide a number of useful pieces of geophysical information, including

(i) the frequency dependence of the Love number $k_2$ due to the dispersion effects and due to core resonances,
(ii) the Earth's specific dissipation function or $Q$ at long periods,
(iii) the nature of energy dissipation in the oceans by providing estimates of the total rate of tidal energy dissipation in the ocean,
(iv) the acceleration of the Moon in its orbit and of dissipation of tidal energy within the Moon,
(v) constraint on electro-magnetic properties and processes at the core-mantle boundary.

The developments up to the late 1970's were examined and reviewed in 1980 (Lambeck, 1980), when developments in space science and technology were spurring new interest in the subject of the Earth's rotation. Precise tracking of satellites for gravity field studies, laser ranging for studying lunar motion, long-baseline radio interferometric observations for deciphering extra-galactic radio sources, and the precise manoeuvring of interplanetary flights all require a precise tracking of the motions of the Earth's rotation axis. At the same time, these new techniques permit the
changes in rotation to be measured with an unprecedented precision and resolution (Lambeck, 1988). The geophysicist's signal is astrophysicist's and space engineer's noise. The most notable achievement with the new technology by that time had been the determination of polar motion from the Doppler tracking of satellites. In 1980, however, the impact of these new methods on the geophysical discussion had been minimal but the results were sufficiently promising to anticipate that new excitation functions would rise out above the measurement noise level.

I will limit the discussion in this Chapter to a few topics where either significant progress has been made since 1980 or where the new observations may lead to new insight into the physical processes, although the time span of the new data is too short, at most six Chandler wobble periods, to expect substantive developments in understanding the Earth's rotation that can be attributed solely to the result of the new methods. Much of the present discussion is, therefore, still based on the older observational data: an appropriate reminder that this earlier data will remain important for many years to come. Brief reviews, in which some of this progress is also discussed, are by Merriam (1983), Lambeck (1987), Rochester (1984), and Wahr (1985).

2. EQUATIONS OF MOTION

The fundamental equation describing the rotation of a non-rigid planet is the Eulerian equation in which both the angular momentum and inertia tensor of the planet and the torques $L$ acting on it are time dependent. These equations are sometimes referred to as the Liouville equations. In most discussions of non-rigid rotation, the excursions from rigid body rotation are small and a perturbation form of the equations suffices. If the conventional terrestrial system, an "earth-fixed" cartesian coordinate system $x_i$ ($i=1,2,3$), is introduced in which $x_3$ lies close to the mean position of the rotation axis and $x_1$ is directed towards the Greenwich meridian, it is convenient to define the rotation of these axes about themselves as

$$
\omega_1 = \omega_0 \, m_1; \quad \omega_2 = \omega_0 \, m_2; \quad \omega_3 = \omega_0 \,(1+m_3);
$$

where $\omega_0$ is the mean angular velocity of the Earth. The $m_i$ are small dimensionless quantities, of the order $10^{-6}$ or less. The $m_1$, $m_2$, $1+m_3$ represent the direction cosines of $\omega$ relative to $x_3$. In particular, $m_1$ and $m_2$ specify the position of the instantaneous rotation axis, the polar motion and, if the $x_1$ axis is directed towards the Greenwich meridian, $m_1$ is the component in the direction $90^\circ$W and $m_2$ is the component in the Greenwich meridian. The third component $m_3$ represents departures in the speed of rotation from the uniform value of $\omega_0$. The change in length-of-day, $\Delta(l.o.d)$ is defined as $m_3 = -\Delta(l.o.d)/(l.o.d)$. The equations of rotation then take the form, with $j = (-1)^j$,

$$
j \, m/\sigma + m = \psi
$$

$$
m_3 = \psi_3,
$$

where
\[ m = m_1 + j m_2 \]  
\[ \psi = \psi_1 + j \psi_2 \]  
\[ \sigma_r = \omega_0 (C-A)/A \] 

C and A are the mean polar and equatorial moments of inertia. The \( \psi_i \) are the excitation functions, characterizing the torques, relative motions and inertia tensor changes. They can be written as

\[ \psi_i = \psi_i(\text{torque}) + \psi_i(\text{matter}) + \psi_i(\text{motion}) \] 

with

\[ \psi(\text{torque}) = jL/\omega_0^2 (C-A) \]  
\[ \psi(\text{matter}) = \Delta I/(C-A) \]  
\[ \psi(\text{motion}) = [\omega_0 h - j \Delta I - j h]\omega_0^2(C-A), \]

and

\[ \psi_3(\text{torque}) = (1/C\omega_0) \int_0^t L_3 \, dt \]  
\[ \psi_3(\text{matter}) = -\Delta I_{33}/C \]  
\[ \psi_3(\text{motion}) = -h_3/\omega_0 C. \]

The inertia tensor \( I_{ij} \) of the Earth is written as

\[ I_{11} = A + \Delta I_{11}(t), \quad I_{22} = A + \Delta I_{22}(t), \]  
\[ I_{33} = C + \Delta I_{33}(t), \quad I_{ij} = \Delta I_{ij} (i \neq j), \] 

where the time dependent perturbations \( \Delta I_{ij} \) are assumed to be small. Also,

\[ \Delta I = \Delta I_{13} + j \Delta I_{23}; \quad h = h_1 + jh_2. \]

The \( h_i \) are the angular momentum terms resulting from motion of particles relative to the \( x_i \) axes. Equations 2 define the motion of the rotation axis relative to the earth-fixed axes \( x_i \). These equations clearly separate the geodetic problem of measuring the rotation (the left-hand-side of 2) from the geophysical problem of evaluating the mechanisms that drive the variable rotation, the excitation functions \( \psi_i \). These equations also separate the polar motion (equation 2a and the definition 5a) from the length-of-day changes (equations 2b and the definition 5b), a separation that is convenient in view of different observational techniques that have been used in the past, to determine the two aspects of the rotational motion. The above equations are generally valid, provided that the excursions of the instantaneous rotation axis from its mean position are small. Otherwise, a more complete description is required in which the squares and products of small quantities must be retained but now the equations lose much of their convenience. At
the present levels of accuracy of both the geodetic and geophysical data the
first order equations, modified if necessary to include the small ellipticity of
the polar motion path resulting from the axial asymmetry of the Earth's
density distribution \((I_{11} \neq I_{22})\), largely suffice. Equation (2) defines one
part of the rotation problem. The other part is the motion of either the
rotation axis or the earth-fixed axes in space. This latter motion can be
expressed in several ways, for example by Euler's kinematic equations. If
we define an inertial reference frame \(X_i\), the position of \(x_i\) relative to it
can be defined by the Euler angles
\[
\alpha_1 = \text{inclination of the } x_1-x_2 \text{ plane on the mean ecliptic},
\]
\[
\alpha_2 = \text{angle in the } X_1-X_2 \text{ plane between } X_1 \text{ and the }
\text{ascending node of the } x_1-x_2 \text{ plane on the ecliptic},
\]
\[
\alpha_3 = \text{angle in the } x_1-x_2 \text{ plane between the ascending }
\text{node and the } x_1 \text{ axis.}
\]
The Euler equations then are
\[
\begin{bmatrix}
\dot{\alpha}_1 \\
\dot{\alpha}_2 \sin \alpha \\
\dot{\alpha}_3 + \dot{\alpha}_2 \cos \alpha_1
\end{bmatrix} =
\begin{bmatrix}
\cos \alpha_3 - \sin \alpha_3 & 0 & 0 \\
\sin \alpha_3 & \cos \alpha_3 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\omega_1 \\
\omega_2 \\
\omega_3
\end{bmatrix}
\] (7)
and they determine the motion of \(x_i\) relative to \(X_i\), once the dynamic
equations (2) have been solved for the \(\omega_i\). In equation (7), the angle \(\alpha_3\)
is equal to \(\omega_3 t\) and has a nearly diurnal period. This means that some nearly
diurnal terms in the solutions for \(\omega_i\), generally of very small amplitude such
as the lunar and solar torques on the earth, become the major terms in the
Eulerian angles \(\alpha_i\). Large-amplitude long-period terms in the solutions for
\(\omega_i\), such as the Chandler wobble, make only very minor contributions to the
\(\alpha_i\). Certain geophysical quantities are therefore best studied by examining the
\(m_i\) (the polar motion and the changes in length-of-day), while others are
best studied by examining the \(\alpha_i\) (the precession and nutation), always
assuming that both astronomical quantities can be measured with comparable
precision. In either case, the geophysical problem is one of evaluating the
excitation functions; the variations with time of the torques \(L\) acting on the
Earth, of the changes in the inertia tensor \(\Delta I\), and of the changes in relative
momenta \(h\) caused by motions of particles making up the Earth relative to
the terrestrial reference axes. An obvious, yet important point is that these
excitations are integral quantities over the whole volume or surface of the
Earth. The Earth's response is a globally integrated one to what may be
either localized or global excitation functions. One example of this is the
periodic change in the mass and angular momentum distribution within the
ocean caused by the tides. The spatial spectrum of these changes is very
broad, yet the Earth's rotation responds only to the small amplitude second
degree component in the tide expansion. As such, the geophysical
constraints imposed by the rotational data are often only of restricted value
in understanding the process themselves.

3. THE FREE ROTATIONAL MODES

The Earth possesses a number of free rotational modes of which the Euler or
Chandler wobble and the nearly-diurnal wobble are geophysically significant. Their observation raises a number of questions. In a physical system such as the Earth, free oscillations are damped and cease to exist with time unless they are continually excited. The persistent occurrence of the Chandler wobble, therefore, raises the question of how this mode is maintained in the presence of dissipation. What is the nature of the excitation? Where is the energy dissipated? The same questions can be raised about the nearly-diurnal wobble except that it has not yet been observed with any certainty. A further question raised by observations of these modes concerns their period. Why does the observed Eulerian wobble period of about 430-435 days differ from that of the predicted values of about 300 days for a rigid Earth or about 270 days for a rigid Earth with fluid core? Part of the answer to these questions lies in the properties characterizing the deformational response of the planet to its rotation, and the study of the free oscillations may be expected to provide global constraints on the elasticity and anelasticity of the mantle and on the dynamics of core motions. Part of the answer also lies in the response of the oceans to the rotation and the study of the free oscillations may also yield some insight into the global ocean dynamics.

Of the two free oscillations discussed here, the Chandler wobble is well observed and its period is generally understood in terms of the departures from rigidity, exemplified by the fluidity of the core, the elasticity and anelasticity of the mantle, and the oceans. The nature of dissipation and excitation are more contentious matters, in part because the two processes are intimately linked. The nearly-diurnal oscillation has been a somewhat esoteric subject because it has remained largely unobserved, but this may no longer be so. Also, because it lies in a frequency band where there are considerable diurnal forced deformations of the Earth, resonance amplification of some of the forced motions may occur and this is anticipated to have observable effects.

### 3.1 THE CHANDLER WOBBLE

#### 3.1.1 General formulation

A qualitative statement of the Chandler wobble problem can be found in most geophysics textbooks (eg Stacey, 1977). Consider a planet whose rotation axis, viewed from body fixed axes, rotates about the principal axis in accordance with Euler's equation. The potential $V_c$ of the centrifugal force at a point $P$ distant $l$ from the instantaneous rotation axis is

$$V_c = \frac{1}{2} \omega^2 l^2$$

where

$$l^2 = \sum \frac{x_i^2}{\omega_i} - \frac{\left( \sum \omega_i x_i / \omega \right)^2}{\omega}$$

This potential can be written as
\[ V_c = \frac{1}{3} \omega^2 r^2 + \Delta V_c \]

where \( r^2 = \Sigma_i x_i^2 \) and \( \omega^2 = \Sigma_i \omega_i^2 \). The second term \( \Delta V_c \) can be expressed in a form

\[ \Delta V_c = \frac{GM}{R^5} \left[ \sum_{m} \left( C_{2m} \cos m\lambda + S_{2m} \sin m\lambda \right) P_{2m}(\sin \varphi) \right] \]  

where

\[ C_{20} = \left( \frac{R^3}{6GM} \right) \left( \omega_1^2 + \omega_2^2 - 2\omega_3^2 \right) \]

\[ C_{21} = - \left( \frac{R^3}{3GM} \right) \omega_3, \quad S_{21} = - \left( \frac{R^3}{3GM} \right) \omega_1 \omega_3 \]  

\[ C_{22} = \left( \frac{R^3}{12GM} \right) \left[ \left( \omega_2^2 - \omega_1^2 \right) \right], \quad S_{22} = - \left( \frac{R^3}{6GM} \right) \omega_1 \omega_2 \]  

The potential \( \Delta V_c \), harmonic in degree 2, deforms the Earth and, for an elastic response, produces an additional potential that can be defined with the aid of a rotational Love number \( k_2 \) as \( k_2 \Delta V_c(R) \) at \( r=R \). The form \( (8a) \) is preserved outside this surface but the coefficients \( C_{2m}^*, S_{2m}^* \) are multiplied by \( k_2 (R/r)^3 \). That is,

\[ \Delta V_c = \frac{GM}{R} \left[ \sum_{m} \left( C_{2m}^* \cos m\lambda + S_{2m}^* \sin m\lambda \right) P_{2m}(\sin \varphi) \right] \]

It is this contribution that modifies the rotational motion of the planet from that described by the Eulerian solution. Equating the \( k_2 C_{2m}^* \) and \( k_2 S_{2m}^* \) with the appropriate elements in the second-degree inertia tensor gives the following time dependent elements \( \Delta I_{ij}(t) \)

\[ \Delta I_{13} \quad \Delta I_{23} = \frac{k_2 R^5 \omega_0^2}{3G} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \]

The rotation excitation functions \( \psi_i \) defined by equation (5a) become

\[ \psi_1 = \frac{k_2}{k_0} \left( m_1 + \frac{m_2}{\omega_0} \right) = \frac{k_2}{k_0} m_1 \]

\[ \psi_2 = \frac{k_2}{k_0} \left( m_2 - \frac{m_1}{\omega_0} \right) = \frac{k_2}{k_0} m_2 \]

with

\[ k_0 = 3(C - A)G/\omega_0^2 R^5 = 3GMc_{20}/\omega_0^2 R^3 = 0.942 \]
With the definition

$$\sigma_0 = \sigma_r \left(1 - k_2/k_0\right), \quad (12)$$

where \(\sigma_r\) is defined by (3c), the equations of motion (2a) with (5a) and (9) are, in the absence of all other excitations,

$$j \dot{m}/\sigma_0 + m = 0 \quad , \quad (13)$$

and their solution is

$$m = m_0 e^{i(\sigma_0 t - \beta_0)} \quad . \quad (14)$$

The motion remains circular but the frequency is reduced from the Eulerian frequency \(\sigma_r\) to \(\sigma_0\). Observations indicate that \(2\pi/\sigma_0 = 435\) days so that \(k_2 = 0.28\). This is close to the value of about 0.29-0.30 observed for the semi-diurnal body tide Love number and this analysis provides a qualitative explanation for the observed lengthening of the period from about 305 days to 435 days. The validity of equating the tidal and Chandler Love numbers is, however, not obvious and the agreement is largely fortuitous. In particular, the degree of coupling of core motions to the mantle is a significant factor in the Earth's overall rotational response but this is not so for the semi-diurnal tidal Love numbers. Also, the Chandler wobble Love number is a function of the ocean response to the centrifugal potential and a more quantitative evaluation, in terms of the planet's physical properties, is required.

If the Earth is subject to a forced excitation the total excitation \(\psi\) can be considered in two parts: the forcing function \(\psi_r\) evaluated as if the Earth were rigid and a second part \(\psi_D\) corresponding to the rotational deformation of the Earth. This is the contribution (10), or \(\psi_D = (k_2/k_0)m\) and the equations of motion now are

$$j \dot{m}/\sigma_0 + m = \left[k_0/(k_0 - k_2)\right] \psi_r \quad . \quad (15)$$

Thus the elastic yielding of the Earth modifies the amplitude of the excitation function by a factor \(k_0/(k_0-k_2)\) and the effect of specific excitation functions can be evaluated as if the planet is rigid and using equation (15) in which \(\sigma_0\) corresponds to the free oscillation frequency of the real Earth. This is appropriate for an excitation that does not load the Earth. If it does then \(\psi_r\), must be replaced by \((1+k_2)\psi_r\), where \(k_2\) is the second degree potential load Love number, and in general,

$$j \dot{m}/\sigma_0 + m = K \psi_r \quad , \quad (16)$$

where the wobble transfer function \(K\) is equal to \(k_0/(k_0-k_2)\) when the excitation does not load the Earth and is equal to \((1+k_2)k_0/(k_0-k_2)\) when it does.
In the presence of linear damping the deformational bulge of the planet lags the potential of the centrifugal force by an angle $\epsilon$ such that
c.f. 9)

$$\frac{[\Delta I_{13}]}{[\Delta I_{23}]} = \frac{k_2 R^2 \omega_0^2}{3G} \begin{bmatrix} \cos \epsilon & \sin \epsilon \\ -\sin \epsilon & \cos \epsilon \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}. \quad (17)$$

For small $\epsilon$, such that $\cos \epsilon \approx 1$ and $\sin \epsilon \approx Q_w^{-1}$,

$$\psi = (k_2/k_0)(1 - jQ_w^{-1})m,$$
leading to the equation of motion

$$\dot{m}/\sigma_r - j\left[1 - \frac{k_2}{k_0}(1 - jQ_w^{-1})\right]m = 0 \quad (18a)$$

whose solution is

$$m(t) = m_0 e^{j(\sigma_0 + j\alpha)} = m_0 e^{j\sigma_0} \quad (18b)$$

with a dissipation time constant $\alpha^{-1}$ given by

$$\alpha = \sigma_0 Q_w^{-1} k_2/(k_0 - k_2) = \sigma_0/2Q_w \quad (18c)$$

The damping parameters $\alpha$ or $Q_w$ serve, like the rotational Love number, as convenient functions to describe qualitatively the rotational motion but, without quantitative evaluations of specific energy sinks, they do not simplify the physics of the problem. Recent observations suggest that $50 \leq Q_w \leq 100$ (Okubo, 1982b) and the damping time constant is about 20-40 years.

In terms of energy dissipation, the wobble $Q_w$ is defined as

$$Q_w^{-1} = \frac{1}{2\pi E_w} \int \frac{dE}{dt} dt = \frac{1}{2\pi} \frac{\Delta E}{E_w} \quad (19a)$$

where $E_w$ is the total wobble energy stored in the Earth that would, in the absence of any excitation, be dissipated in the time interval $0 \leq t < \infty$ and includes the kinetic strain, gravitational and ocean pole tide energies. $\Delta E$ is the amount of energy dissipated in one cycle of the motion. It should be noted that $Q_w$ is not directly comparable with the shear mantle $Q_\mu$ as used in seismology. Several recent studies (Smith and Dahlen, 1981; Ben Menaham, 1982; and Okubo, 1982b) obtained

$$Q_w/Q_\mu = 1.5 - 2.1 \quad (19b)$$

There are two ways in which the excitation mechanisms can be tested. One is to compute a theoretical $m(t)$ for a specified $\psi(t)$; the other one is to deconvolve the observed $m(t)$ to estimate an "observed" $\psi(t)$ and to
compare this with the model value. The deconvolution process is one of estimating an "observed" $\psi(t)$, from the observed $m(t)$ that contains a noise component. A simple process that does not attempt to separate noise from the signal $m(t)$ is to write, for a discrete observation series, for time $t$

$$m(t) = e^{j\sigma_0 t} \left[ m_0 - j\sigma_0 \sum_0^t \psi(\tau) e^{-j\sigma_0 \tau} \Delta t \right]$$

and for an instant $t-\Delta t$

$$m(t-\Delta t) = e^{j\sigma_0 (t-\Delta t)} \left[ m_0 - j\sigma_0 \sum_0^{t-\Delta t} \psi(\tau) e^{-j\sigma_0 \tau} \Delta t \right].$$

Then

$$m(t) = m(t-\Delta t)e^{j\sigma_0 \Delta t} - j\sigma_0 \psi(t) \Delta t,$$

and the inferred excitation is

$$\psi^0(t) \equiv \psi(t) = - (1/j\sigma_0 \Delta t)[m(t) - e^{j\sigma_0 \Delta t} m(t-\Delta t)].$$

More sophisticated deconvolution filters have been proposed (see, for example, Smylie et al., 1970; Wilson and Haubrich, 1976).

The comparisons can also be made in the frequency domain. A series of step-function excitations will produce a spectrum of $\psi(t)$ that is "red", one of decreasing power with increasing frequency, whereas a series of delta-function excitations will produce a "white" or flat spectrum. It becomes possible, in theory at least, to gain insight into the forcing process by examining the excitation series derived by a deconvolution process from the polar motion process. Evaluation of the excitation spectra are generally suggestive of a red noise process, with power increasing towards the low frequencies, but in the neighbourhood of the Chandler wobble frequency the spectrum is essentially white and consistent with delta-function-like excitations (e.g. Wilson and Haubrich, 1976).

### 3.1.2 Observational evidence for wobble parameters

Numerous analyses of the Chandler wobble parameters have been made in recent years using different data sets and data lengths, different analytical techniques and different assumptions about the statistical nature of the noise of the data and of the excitation process. The estimation problem is one of matching the observed wobble spectrum, containing measurement noise, with the spectrum of the model for the wobble impulse response, usually assumed to be a damped linear oscillator, that has been subjected to an excitation process of certain assumed physical or statistical properties. In the presence of measurement noise $\epsilon_m(t)$, equation (20a) becomes

$$m(t) - e^{j\sigma_0 \Delta t} m(t-\Delta t) = - j\sigma_0 \psi(t) \Delta t + \epsilon_m(t) - \epsilon(t-\Delta t)e^{j\sigma_0 \Delta t}. \quad (21)$$
If the spectra of both $\psi$ and $\epsilon_m$, denoted by $S(\omega)$ and $E_m(\omega)$ respectively, are assumed to be white, such that $S(\omega) = v_\psi^2 = \text{constant}$ and $E_m(\omega) = v_m^2 = \text{constant}$, then the amplitude spectrum of $m$ is

$$S_m(\omega) = \frac{1}{2} \left[ \frac{v_m^2 + \frac{(\sigma\Delta t)^2 v_\psi^2}{1 - 2e^{-\alpha\Delta t} \cos((\sigma - \omega_0)\Delta t) + e^{-2\alpha\Delta t}}} {\Delta t} \right]$$

By matching this spectrum with the observed spectrum it becomes possible to estimate $v_m^2$, $v_\psi^2$, the damping constant $\alpha$ (or $Q_w$) and the free oscillation frequency $\omega_0$. Equation (22) demonstrates clearly the interdependence between the various parameters. The damping parameter $\alpha$ is particularly sensitive to the assumptions made for the noise and excitation spectra. The various published solutions for these parameters differ essentially in the manner in which the observed spectrum is computed or in the manner in which the theoretical spectrum is matched to the observed spectrum.

<table>
<thead>
<tr>
<th>Author</th>
<th>Data</th>
<th>Interval</th>
<th>Period range (sid.d)</th>
<th>$Q_w$ (sid.d)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IPMS</td>
<td>1967-1977</td>
<td>431.39 (446.65 - 416.19)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DMA</td>
<td>1969-1979</td>
<td>431.90 (451.05 - 413.84)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ILS*</td>
<td>1900-1979</td>
<td>434.46 (437.57 - 430.88)</td>
<td>&gt;50 (175)</td>
</tr>
<tr>
<td>Ooe (1978)</td>
<td>ILS</td>
<td>1900-1975</td>
<td>436.01 (434.00 - 438.05)</td>
<td>50-300</td>
</tr>
<tr>
<td></td>
<td>ILS*</td>
<td>1962-1980</td>
<td></td>
<td>50-100</td>
</tr>
<tr>
<td></td>
<td>BIH &amp; IPMS</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 Summary of recent estimates of Chandler period and $Q_w$. The range of period estimates corresponds to ±1 standard deviation. The $Q$ value preferred by Wilson and Vicente is 175 but values greater than 50 are consistent with their data and analysis technique. The asterisk denotes the revised ILS data set.

Table 1 summarizes some recent results for the wobble period and $Q$. The various estimates for this period are in reasonably good agreement. The data sources are the Bureau International de l'Heure (BIH), the International Polar Motion Service (IPMS) and the satellite Doppler solutions by the Defence Mapping Agency (DMA) are consistent (Wilson and Vicente, 1981) although the shorter data sets do result in rather large standard deviations for the period estimates. The revised International Latitude Service (ILS) data set (Yumi and Yokoyama, 1980) gives an estimate of
considerably greater precision than the earlier solutions but the result is consistent with the analyses by Wilson and Haubrich (1976) and Ooe (1978) of an earlier ILS data set. The polar motion results from laser tracking of satellites or interferometric observations of radio sources do not yet cover a long enough time span to be useful.

Estimates of \( Q_w \) are more variable, being critically dependent on the choice of filter parameters used in estimating the power spectrum of the polar motion. Most analyses indicate that \( 50 \leq Q_w \leq 100 \) (Okubo, 1982b; Wilson and Haubrich, 1976; Ooe, 1978) although Wilson and Vicente (1981) obtained \( Q_w = 175 \) from an analysis of the revised ILS data set. Of some concern is the observation by Okubo (1982a) that an analysis of the ILS data for the years after 1963 produces quite different results than either the BIH or IPMS data sets for the corresponding period, placing doubt on \( Q_w \) values obtained from any part of the ILS data set. The estimates of the power in the neighbourhood of the Chandler wobble frequency required to excite the wobble is estimated by Wilson and Vicente (1980) as about \((7 - 8) \times 10^{-8} \) radian\(^2\)/cycles/year and by Ooe (1978) as about \( 21 \times 10^{-8} \) radian\(^2\)/cycles/year, the difference being a consequence of different assumptions made about the measurement noise of \( m \).

3.1.3 Period

The quantitative evaluation of the Chandler wobble period involves the computation of the time dependence of the inertia tensor \( \Delta I_{ij} \) and of the relative angular momentum vector \( \mathbf{h}_i \) associated with the deformation and motion of the mantle, core and oceans. Thus if a particle at its equilibrium position \( x_i \) is moved to a position \( x_i + d_i(x,t) \) in response to the rotation, the time dependent part of the inertia tensor (6a) is

\[
\Delta I_{ij}(t) = \int \rho(x)[2x_k \delta_{ij} d_i(x,t)x_k - x_i d_i(x,t) - d_j(x,t)x_j] dV \tag{23a}
\]

and the relative angular momenta are

\[
\mathbf{h}(t) = \int \rho(x)(x+d)(\partial d/\partial t) dV \tag{23b}
\]

The elastic displacements within the mantle and core follow from the solution of equations that describe the deformation of a self-gravitating, elastic and, in this case, an ellipsoidally shaped rotating planet. An approximate solution follows from the Kelvin-Hough model for the core as defined by equations and from considering the motion relative to Tisserand axes such that \( h_i(\text{mantle}) = 0 \). The mantle's deformational response to the variable rotation is assumed to be linear in \( m_i \) so that the displacements \( d_j \) of the mantle, and therefore the \( \Delta I_{ij} \), are also linear in \( m_i \). In a most general form,

\[
\Delta I_{ij}(t) = D_{ijk}(x,t) m_k(t),
\]

where the \( D_{ijk} \) is a third order tensor and is a function of the mantle parameters. For an axially symmetric Earth, and ignoring the ellipticity of the planet, \( D_{ijk} \) is given simply by (9) or
THE EARTH'S VARIABLE ROTATION: SOME GEOPHYSICAL CAUSES

\[ D = k_2 R^5 \frac{\omega_0}{G} \]

where \( k_2 \) is the conventional static Love number of degree 2 (see also Sasao et al., 1977; Smith and Dahlen, 1981). The equations of motion can then be written as

\[ \dot{m} - i \omega_0 \frac{(C-A)}{A_m} \left[ 1 - \frac{D}{C-A} \right] m - i \omega_0 \epsilon_c \frac{A}{A_m} n = 0 \]  

(24)

\[ \dot{n} + i \omega_0 \frac{(C-A)}{A_m} \left[ 1 - \frac{D}{C-A} \right] m + i \omega_0 \left[ 1 + \frac{A}{A_m} \epsilon_c \right] n = 0 \]

and the corresponding eigenfrequencies are

\[ \sigma_1 \approx \frac{(C-A-D)}{A_m} \omega_0 (1 + \epsilon_c) \left[ 1 - \frac{A}{A_m} \epsilon_c \cdot \frac{(C-A-D)}{A_m} \right] = \sigma_e \]  

(25a)

\[ \sigma_2 \approx - \omega_0 \left[ 1 + \frac{A}{A_m} \epsilon_c - \frac{(C-A-D)}{A_m} \epsilon_c \right] \approx \omega_0 (1 + \frac{A}{A_m} \epsilon_c) \]

The first is the Chandler wobble frequency, corrected for the mantle elasticity and the ellipsoidal core. The second is a nearly-diurnal wobble. The radial density structure of the entire Earth is used to compute the elastic yielding factor \( D \) but for evaluating the fluid core effect the core has been assumed to be of constant density. These solutions do not specifically allow for the elastic deformation of the core-mantle boundary, a neglect that is most severe for the second oscillation of frequency \( \sigma_2 \). More rigorous treatments are given by Smith (1977) and Sasao et al. (1980). These last authors obtained

\[ \sigma_2 = - \omega_0 \left[ 1 + \frac{A}{A_m} (\epsilon_c - \beta) \right] \]  

(25b)

where \( \beta = 6 \times 10^{-4} \) is a parameter defining the deformation of the mantle and core, and the ellipticity of the core-mantle boundary is effectively reduced by about 25%, because of the deformation.

Of the constants in (25a), \( C \) and \( A \) are based on observations of the precession constant \( H \) and the dynamic flattening and are known with sufficient precision, and

\[ C = 8.0394 \times 10^{37} \text{ kg m}^2 \]  

(26a)

\[ A = 8.0131 \times 10^{37} \text{ kg m}^2 \]

The core and mantle inertia elements are estimated from Earth density models based on the inversion of seismic data and on the assumption of hydrostatic equilibrium. For Earth model 1066A of Gilbert and Dziewonski (1975), Smith and Dahlen (1981) obtained
Also, for the same model, \( k_2 = 0.30088 \). With these values the wobble period is 396.4 sidereal days. The more precise theory by Smith and Dahlen gives 396.9 days. Some uncertainty in this value results from the limitations of the Earth model, about \( \pm 0.5 \) days according to Smith and Dahlen (see also Sasao et al. 1980, Table 1, for estimates of this period for different Earth models). A further uncertainty arises from the assumption that the planet is in hydrostatic equilibrium, through the estimates of the core and mantle inertia elements, something that the planet is manifestly not.

Variations in the Earth's rotation induce small amplitude ocean tides. Of particular interest is the so-called "pole tide". This oceanographic curiosity is of more than passing geophysical interest because it significantly modifies the Earth's wobble from what it would be for an oceanless planet, and it may also account for the damping of the wobble. The amplitude of the pole tide is at most a few millimeters and only occasionally rises above the noise level of the sea level observations. Models of this tide are therefore, based on theoretical arguments, the simplest of which is that the tide follows an equilibrium theory. By the nature of the ocean-continent distribution the tide exhibits more regional variability than does the driving potential but only the components of degree 2 and order 1, those contributing to the inertia elements \( \Delta I_{13} \) and \( \Delta I_{23} \), are relevant here. This abbreviated tide is given by

\[
\xi = -\left( R^2 \omega_0^2 / 6g \right) (1 + k_2 - h_2) \left[ (A_1 m_1 + A_2 m_2) \cos \lambda + (B_1 m_1 + B_2 m_2) \sin \lambda \right] P_{21}(\sin \varphi),
\]

where the \( A_i \) and \( B_i \) are functions of the coefficients in the ocean expansion (Lambeck, 1980, p.143). The time-dependent inertia tensor follows directly and the free oscillation frequency of the Earth is modified to, in a first approximation,

\[
\sigma_0 = \sigma_e \left( 1 - \frac{1}{2} \omega_0 \frac{\psi_0}{\sigma_e} (A_1 + B_2) \right).
\]

where \( \sigma_e \) is given by (25a). With \( (A_1 + B_2) = 3.05 \) and with \( \sigma_e = 2\pi/396.9 \) days

\[
\sigma_0 = 0.936 \sigma_e.
\]

The Chandler motion is no longer circular but the ellipticity is small, the ratio of the two axes differing from unity by only about 0.3%. The effect of the equilibrium pole tide is to lengthen the wobble period by 27.2 days, from 396.9 days for the elastic body to 424.2 days for the elastic body with an equilibrium ocean tide. Smith and Dahlen (1981) obtained an increase of 29.8 days and Dickman and Steinberg (1986) obtained 30.9 days using more rigorous descriptions of the mantle deformation effects of the ocean tide.
Does the pole tide follow an equilibrium theory? The answer to this question is clearly of geophysical importance for if departures from equilibrium are significant, yet impossible to evaluate with precision, little or no information can be gleaned about the anelasticity of the Earth's mantle at the wobble frequency. On the other hand, if departures from equilibrium can be established then this contributes to the knowledge of the dynamics of long-period tides.

One consequence of a non-equilibrium tide is that the ocean surface deformation lags behind the rotation axis (c.f. equation 17) but for plausible values of this lag the period of the wobble is not affected by more than a fraction of a day. Alternatively, the tidal amplitudes could be enhanced, as would occur if the pole tide is near a resonance in the ocean and then \( \sigma_0 = \sigma_0(1-\delta \Delta \sigma) \) where \( \Delta \sigma \) is the equilibrium ocean effect on the wobble frequency and \( \delta \) is the amplification factor. The enhancement required to explain the discrepancy of about 8 days between the observed period of 435 days and the above model period of about 426-427 days is, from (28a) about 1.3.

Significant local enhancement of the pole tide has been known to occur but observational evidence for this tide in the open oceans remains scant. Certainly the requisite \( \delta \) factor of 1.3 cannot be ruled out from the observational evidence. Theoretical arguments, however, run mostly in favour of a pole tide that closely follows equilibrium theory (see the recent studies by O'Connor and Starr, 1983; Dickman, 1985; Carton and Wahr, 1986; O'Connor, 1986; Wunsch, 1986). Wunsch argues that any departures from equilibrium in the open ocean will be too small to drive the observed pole tide in the North and Baltic Seas and that the observed enhancement must be the result of a near resonance of long-period waves in the basin with the pole tide potential. One exception to the above conclusion is the study by Molodenskiy (1985) who developed a more complete pole-tide model. Some of the previous authors developed this tide for either global oceans or for oceans with a zonal geometry and here the tide effectively follows an equilibrium theory but, according to Molodenskiy, for realistic ocean geometries the departures from equilibrium become significant. This conclusion does not follow from the results of Carton and Wahr (1986) and O'Connor (1986) for models of the tide in restricted ocean basins, and confirmation of Molodenskiy's results is desirable.

This uncertainty about the non-equilibrium pole tide is unfortunate for it does not permit strong conclusions to be drawn about the alternate explanation, that the discrepancy in period is a result of the anelasticity of the mantle. If it is, then \( \sigma_e \) in equation (28a) must be written as \( (\sigma_e + \Delta \sigma) \), with the requirement that the \( \sigma_0 \) in this equation equals the observed value. With the Smith and Dahlen value for the equilibrium ocean tide lengthening of the period, the required decrease in frequency is \( \Delta \sigma = -0.0176 \sigma_e \). Any such anelastic lengthening of the wobble period can be described by a modified Love number at frequency \( \sigma \) as

\[
k_2(\sigma) = k_2 + \Delta k_2
\]

where, \( k_2 \) is the elastic value. For the wobble model (5),
\[ \Delta k_2 = - \left( k_0 / \sigma_F \right) \Delta \sigma \approx 0.013. \]

This is the increase in the Love number \( k_2 \) over the value based on seismic parameters. A popular \( Q \) model in \( Q = Q_0(frequency)^{\gamma} \), with \( \gamma \approx 0.3 \), where \( Q_0 \) is the \( Q \) value at the reference frequency, such as the frequency of the second degree free seismic oscillation (e.g. Anderson and Minster, 1979). With a mantle \( Q \) of between 350 and 600 at 54 minutes, \( 0.18 < \gamma < 0.30 \). More precise models of this dispersion effect, based on realistic Earth models rather than on the Kelvin model, have been discussed by Smith and Dahlen (1981); Okubo (1982b) and Dehant (1987) with essentially similar results.

The ultimate estimates for the frequency dependence of attenuation in the Earth from the Chandler wobble period are only as reliable as is the assumption that the discrepancy in the wobble period of 7-8 days can wholly, or in a known part, be attributed to mantle anelasticity; that the pole tide can be adequately modelled. This question is an important one because while considerable information on the Earth's \( Q \) exists at seismic frequencies, the long period \( Q \)'s are poorly constrained and the validity of extrapolating \( Q \) models beyond the seismic frequencies remains an open question.

3.1.4 Dissipation

The second aspect of the Chandler wobble is the dissipation process: where is the wobble energy dissipated? Is it in the oceans or mantle, or is it at the core-mantle boundary? This last possibility is usually discounted as estimates of viscous dissipation and electromagnetic damping appear to be wholly inadequate. The wobble dissipation function, or \( Q_W \), is defined by equation (19a). Statistical analyses of the observations of the polar motion lead to estimates of \( Q_W \) and the geophysical problem is to evaluate both the total wobble energy stored in the Earth during one cycle of the oscillation and the rate at which this energy is being dissipated.

An order of magnitude of the rate of energy dissipation follows from considerations of the kinetic energy only. In its initial state the kinetic energy is

\[ E_i = \frac{1}{2} A (w_1^2 + w_2^2) + C w_3^2 \]

and when the wobble is completely damped the final kinetic energy is

\[ E_f = \frac{1}{2} C w_2^2 \]

with

\[ A^2 (w_1^2 + w_2^2) + C^2 w_3^2 = C^2 w_2^2 \]

because angular momentum is conserved. The loss of energy is, with (18b)

\[ E = E_i - E_f = \frac{1}{2} A A / \omega^2 m_0^2 e^{-2\sigma t} \]
where $H$ is the precession constant. The rate of energy dissipation is

$$\frac{dE}{dt} = \frac{g a_0}{2 \Omega_w^2} A H \omega^2 m_0^2 e^{-\sigma_0 t/\Omega_w} = 10^6 \text{ W.}$$

(29)

This is about six orders of magnitude less than the rate of energy dissipation in the semi-diurnal ocean tide.

Perhaps because of the insignificance of this energy loss, the sink has not been unambiguously identified. Ocean dissipation has generally been thought to be unimportant and this is perhaps surprising in view of the importance of the oceanic dissipation at the higher tidal frequencies. This wobble energy source can be evaluated in a similar way as the semi-diurnal and diurnal tides. The abbreviated pole tide is given by (34) and the dissipation can be modelled by introducing a lag $\epsilon_0$ in this response. The resulting equations yield a relaxation time constant of

$$\alpha = \omega_0 (\sigma_0/\sigma_e) \epsilon_0 \psi_0 (A_1 + B_2)/2$$

(30a)

and

$$Q_w^{-1} = \epsilon_0 \psi_0 (A_1 + B_2) (\omega_0/\sigma_e)$$

(30b)

For $Q_w = 1090$, $\epsilon_0 = 4^\circ$. Published information on the lag of the pole tide is even more sparse than the amplitude information. Values of a few tens of degrees have been reported from regions where the tide has been locally enhanced but global values cannot be established.

Wunsch (1974) assumes that the dissipation is by bottom friction in shallow seas and he estimates a dissipation of about $8 \times 10^4$ W for the North Sea although he cautions that this value is extremely model dependent and that this estimate could be excessive by an order of magnitude (see also Wunsch, 1986; Dickman and Preisig, 1986). Extrapolation to all of the world's shallow seas leads to a total dissipation of $4 \times 10^6$ W and possibly as low as $4 \times 10^5$ W, but, again, Wunsch cautions that these estimates may be excessive because some of the conditions that could have combined to produce the enhanced pole tide in the North Sea may not occur everywhere. If the bottom friction model is valid, then the pole tide currents in these restricted regions need to influence the global pole tide so as to produce the requisite lag angle. Perhaps this can be tested by numerical models. Other processes that lead to a more uniform dissipation in the oceans, either by dissipating energy along the coastlines, such as the Proudman model or internal friction, must also be considered.

If dissipation occurs primarily in the mantle then the shear $Q_{\mu\nu}$ corresponding to a wobble $Q$ of 70–200, lies in the range 30–100 (equation 19b). This compares with a value of about 350–600 for the spheroidal seismic free oscillation mode of degree 2 whose period is about 54 minutes and this is suggestive of a strong frequency dependence. With the above frequency dependent $Q$ law this leads to $0.18 < \gamma < 0.30$, (see also Smith and Dahlen, 1981; Molodenskiy and Zharkov, 1982; Okubo, 1982b) and this is consistent with the result obtained above from the analysis of the period elongation. However, the result is only as good as the assumption that
dissipation occurs only in the mantle. The whole problem of dissipation remains wide open.

3.1.5 Excitation

The study of the excitation of the Chandler wobble by the atmosphere goes back to work by V. Volterra in 1895. The essential argument is that the atmospheric pressure variation over the Earth's surface is not strictly annual and that the wobble is maintained by the irregular variation in the elements in the excitation function. The problem is one of evaluating the excitation due to atmospheric motion and mass transport and to compare either the motion of the pole driven by this excitation with the observed motion, or the power in the excitation spectrum with the power deduced from the astronomical observations. This atmospheric forcing function is discussed further below. It contains two parts, \( \psi(\text{motion}) \) and \( \psi(\text{matter}) \). The first requires an evaluation of the relative angular momenta \((h_1, h_2)\) from global meridional winds at all altitudes. Alternatively, the wind effect can be evaluated by computing \( \psi(\text{torque}) \) and this requires a knowledge of surface winds, of surface friction coefficients and of surface topography. \( \psi(\text{motion}) \) is generally believed to be small. This is indeed fortunate because measurements of the temporal and spatial variation of meridional winds are relatively sparse, and estimates of surface friction coefficients remain unreliable. The evaluation of \( \psi(\text{mass}) \) requires information of surface pressure only and it is usually easier to estimate this function than it is to estimate \( \psi(\text{motion}) \) from available meteorological observations.

A number of estimates of \( \psi(\text{mass}) \) have been made since Volterra's original suggestion. These include studies by Jeffreys in 1940, Munk and Hassan in 1961, Wilson and Haubrich (1976) and Wahr (1983). The general conclusion reached in these last three studies, based on relatively complete global monthly mean surface pressure variations for the same length of record as the astronomical time series, is that while there is significant power in the excitation spectrum it is insufficient, by a factor of 2 or 3, to maintain the wobble against damping. Wilson and Haubrich (1976) have argued that \( \psi(\text{motion}) \) from mountain torques does make a significant contribution but other studies do not support this conclusion.

In recent years, high resolution wind and surface pressure data has become available as part of global meteorological observing programs such as the Global Atmospheric Research Program and detailed excitation functions have been computed at daily intervals (Barnes et al., 1983). These data sets may lead to a revision of the atmospheric excitation hypothesis if changes in \( \psi \) occur on time scales that are much less than a month but, at present, these studies cover intervals that are too short compared with the damping time constant to estimate whether the motion can be maintained against dissipation of the wobble energy.

It has recently been suggested that variations in the ground water storage, including ground water, and water stored on the surface as ice and snow, may be sufficiently variable to contribute to the wobble excitation (Hinnov and Wilson, 1987). This hydrologic excitation function is somewhat smaller in magnitude than the seasonal atmospheric excitation of the polar motion (see Lambeck, 1980; p.154) but, because of the seasonal patterns of snow and ice coverage, the spectrum may contain significant power over a
broad frequency band centred on the 12 month period (see also Chao et al., 1987). Global time series of hydrologic parameters are, however, incomplete and satisfactory tests of this hypothesis are difficult to make. Spectral analyses of the polar motion indicate that the power about the annual frequency is concentrated in a very narrow frequency band and this seems to argue against the hypothesis that major departures occur in the hydrologic excitation function from a strictly sinusoidal oscillation.

Soon after Chandler's discovery, J. Milne suggested that there may be some relation between polar motion and seismic activity. Later, in 1928, G. Cecchine also suggested such a relationship and the hypothesis that the wobble is excited by changes in the Earth's inertia tensor caused by large earthquakes has not been far from the forefront of discussions of the wobble ever since. If the earthquake modifies the inertia tensor according to a step-function at time $t_s$, no instantaneous change in the position of the rotation pole occurs and there is only a change in the direction of the pole path. A succession of earthquakes, associated with sufficiently large changes in the excitation function could then maintain the wobble and explain, at the same time, any secular drift in the pole position. That is (Lambeck, 1980)

$$m(t) = m_0 e^{i\omega t} + \frac{\omega}{\sigma_0} \sum \Delta I_{t_s} e^{-i\omega t} + \frac{\omega}{\sigma_0} \sum \Delta I_{t_s} e^{-i\omega t}$$

where the summation is carried out over all events occurring at times $t_s$. The second term is the shift in position of the excitation pole or of the mean pole of rotation while the third term represents the modified Chandler wobble about this new position of the mean pole. If the basic assumption of the step function model is valid, it remains to evaluate the $\Delta I$ from parameters characterizing the earthquake displacement field. Several authors have derived equations for this (Smylie and Mansinha, 1971; Dahlen, 1973; Israel et al., 1973; Mansinha et al. 1979) using elastic dislocation theory. The resulting expressions are of the form

$$\Delta I_{13} + j\Delta I_{23} = M_0 \sum_{i=1}^{3} \Gamma_i (r)(g_i + jh_i)$$

where $M_0$ refers to the seismic moment of the earthquake, $\Gamma_i$ are functions of geocentric distance and the radial variations of elastic moduli and density and the $g_i$ and $h_i$ are functions of the earthquake coordinates and parameters defining the orientation of the fault plane and the direction of motion. A different approach has been adopted by O'Connell and Dziewonski (1976) who expressed the displacements in terms of a normal mode expansion but the results obtained by the two methods are consistent (see Table 8.2 of Lambeck, 1980). All of these studies indicate that very large earthquakes, such as the Chile (1960) and Alaska (1964) earthquakes, are able to shift the mean pole by amounts of 0°01 to 0°02.

The major difficulty with testing the seismic excitation hypothesis is the estimation of realistic seismic moments for most of the earthquakes. $M_0$ can be estimated from long-period spectra of the earthquake waves or from empirical relationships with the short-period seismic magnitude. Most
of the moments of the older earthquakes, prior to about 1960, are based on
the latter approach but different laws produce quite different moments once
the magnitudes exceed about 7.5. There simply have not been enough very
large earthquakes to permit a reliable average relationship to be established
and considerable debate remains about the size of the moments of the large
earthquakes.

The above shifts of $0.01 - 0.02$ are for the two largest moment
earthquakes of this century but there have not been enough comparable
earthquakes before or after these two events to maintain a wobble that
exhibits little evidence for major attenuation. Of earthquakes of recent
years for which precise seismic moments are available, none have been
adequate to maintain the motion against damping (Souriau and Cazenave,
1985; Gross, 1986) and just as the accuracy of the polar motion observations
has increased significantly, so have the estimates of the seismic moments
decreased! It has been argued that aseismic deformations, occurring at
periods that are short compared with the wobble period but which lie
outside the bandwidth of most seismometers, account for the missing
excitation. Evidence for the aseismic deformations includes observations of
slow (by seismology standards) deformation prior to the major shock.
Studies of the after-shock area also provide evidence for post-seismic
deformation: estimates of the fault plane areas based on after-shocks several
months after the main shock are often much larger than the areas estimated
from after-shocks immediately following the earthquake. The general
discrepancies between estimates of seismic slip and plate motions along many
plate boundaries, also suggests that significant pre- and post-seismic
deformations occurs. Yet the observational evidence remains inadequate for
evaluating quantitatively the role of aseismic deformation in exciting the
Chandler wobble.

A third possible excitation mechanism is the action of
electromagnetic torques on the base of the mantle. These torques arise from
the penetration into the lower mantle of time-dependent magnetic fields
self-generated within the liquid core, and they have often been invoked to
explain the decade changes in the length-of-day (see below). It has been
suggested by Runcorn (1982) that these torques also excite the Chandler
wobble.

For the purpose of calculation, a stationary dipole magnetic field
$B$ is assumed to exist within the core and mantle. Differential motion
between the highly conductive core and less conductive mantle is also
assumed to occur. This relative movement produces an electric current
density $J$ that penetrates into the imperfectly insulating lower mantle. There
it interacts with the field $B$ to produce the Lorentz force $J \land B$ and a
torque on the mantle of (Rochester, 1962)

$$L = \int_{V_m} r \land (J \land B) \, dV \quad .$$

(32)

The integral is over the conducting part of the mantle. The geophysical
problem lies in the evaluation of the current $J$ circulating in the lower
mantle. Dynamo theories generally predict the existence of a strong toroidal
field in the core, produced by the differential movements at both the inner
core - outer core - mantle boundaries, and it is the current produced by this part of the field that produces a net torque on the mantle. But this toroidal field is obscured from observations by the low electrical conductivity of much of the upper mantle and any estimates of the efficiency of the electromagnetic coupling are strongly model dependent. For these torques to excite the wobble they must occur over an interval of time that is short compared to the wobble period.

Two time constants control this coupling (Roberts, 1972). The first, \( \tau_1 \), governs the time with which sudden changes in the toroidal field penetrate into the mantle. The second, \( \tau_2 \), is the time constant of the electromechanical coupling. Neither \( \tau_1 \) nor \( \tau_2 \) alone is appropriate for characterizing the time constant for coupling but Roberts showed that the combination \( \tau_3 = \frac{\tau_1}{3} \frac{\tau_2^2}{3} \) provides a reasonable estimate. A simple model is provided by Roberts in which

\[
\tau_1 = \mu_0 \frac{\langle \lambda > L^2}{2}, \quad \tau_2 = \frac{C_m C_c}{C} \frac{1}{4\pi \langle \lambda > a_c^4 L B_r^2}.
\]

(33a)

In these equations \( \mu_0 \) is the magnetic permeability and is nearly equal to that of a vacuum or \( 4\pi \times 10^{-7} \) H m\(^{-1} \). \( \gamma \) is the electrical conductivity, and \( \langle \lambda > \) is the average value for of the lower mantle, of thickness \( L \), defined as

\[
\langle \lambda > = \frac{a_c + L}{a_c} \int_a^b \lambda \, dr
\]

(33b)

where \( a_c \) is the radius of the core. \( B_r \) in (33a)) is the mean value of the vertical field at the core-mantle boundary. More complex models are discussed by Braginskiy and Fishman (1976) and LeMouel and Courtillot (1982) (see also Rochester, 1984). The evaluation of the models requires a knowledge of the lower mantle conductivity and of the strength of the field \( B_r \) at \( a_c \). The latter can be approximately estimated by extrapolating the surface field downwards and \( B_r(a_c) = 3.5 \times 10^{-4} \) T. The conductivity of the lower mantle is poorly constrained (see, for example, Figure 8.5 of Stacey, 1977) and average values for the lower mantle range from about 100 \( \Omega^{-1} \) m\(^{-1} \) with \( L = 100 \) km (McDonald, 1957) to about 400 \( \Omega^{-1} \) m\(^{-1} \) with \( L = 2000 \) km (Backus, 1983). Then \( \tau_3 \) ranges from about 8 years for the upper value to about 15 years for the lower value. The more complex model of Braginskiy and Fishman (1976) gives \( \tau_3 = 8 \) years for the conductivity model of Backus (Rochester, 1984) and all models give time constants that are more than an order of magnitude greater than required to excite the wobble. A principal limitation of these models is that they account for only the lowest-order terms in the magnetic field so that the actual field strength at the core-mantle boundary may be quite different than that assumed. Also, fluctuations with periods less than the screening time constants are not observed at the surface and, if they occur, they could significantly enhance the coupling (Hide, 1966).

There does appear to be another way to test the hypothesis. Any electromagnetic torques will also excite the length-of-day changes for there is no obvious reason why the torques should be primarily meridional. It is
possible, therefore, to also test the hypothesis by examining the spectrum of length-of-day changes. Runcorn suggests that the impulse torque required to produce a change in the radius of the pole path of 0.01 is about equal to that required to produce the observed decade fluctuations, of 5\times10^{-8}, recorded in the proportional change in length-of-day or \( m_3 \). The problem is one of time scale: the \( m_3 \) power spectrum at periods of about 5 years and less is considerably smaller than at periods of the order of decades (Morrison, 1979), particularly when the \( m_3 \) have been corrected for the known atmospheric excitation (Lambeck and Hopgood, 1982) and it does not appear that such an impulse, occurring at intervals of about a decade or longer, can maintain the wobble against significant damping. The argument for exciting the Chandler wobble by these electromagnetic torques is, therefore, not strong.

The question of the nature of the excitation of the Chandler remains open. No single mechanism appears to be adequate to excite the wobble indefinitely and a combination of seismic and aseismic, atmospheric and hydrologic, and magnetic forcing functions is likely to be required. Yet not any one of these complementary contributions can be evaluated with sufficient confidence to permit the other mechanisms to be tested. What is required here is not so much improved astronomical or geodetic observations as improved geophysical models and observations to permit the excitation functions to be realistically estimated.

3.2 THE NEARLY-DIURNAL Wobble

The frequency of the nearly-diurnal wobble of the Kelvin-Hough model is given by

\[
\nu_2 = -\omega_0 \left[ (1 + \frac{A}{\lambda_\text{m}} \epsilon_C) + \mathcal{O}(\epsilon_C^2) \right],
\]

where \( \epsilon_C \) is the flattening of the core-mantle boundary. For an Earth in hydrostatic equilibrium, with the parameters (26), and \( \epsilon_C = 1/392.7 \), \( \sigma_2 = -1.003 \) (sidereal days)^{-1} and the period is about 23 hr 52 m. The effect of the mantle elasticity and realistic density distributions in the core is given by equation (25b) and these more realistic models give \( \sigma_2 = -1.0022 \) (sidereal days)^{-1} (Smith, 1977; Sasao et al. 1980). According to Sasao et al. the adoption of different core and mantle models modifies \( \sigma_2 \) by less than 10^{-4}. The corresponding period is 23 hr 52 m 55(\pm 5)s. While this frequency is relatively insensitive to the details of the Earth model it is more sensitive to the choice of core-mantle boundary geometry. With

\[
d\epsilon_C = (d\epsilon_C - d\epsilon_C)/a_C,
\]

where \( a_C, \epsilon_C \) are the equatorial and polar radii of the core. For a departure of this boundary from equilibrium by \( \pm 1 \) km \( d\sigma_2 = 4\times10^{-4} \omega_0 \) and the change in period is about \( \pm 30 \) seconds.

The existence of the free wobble for the Kelvin-Hough model has long been recognized and the qualitative agreement between this result and the observed Chandler wobble suggests that the Earth should also possess the core mode, always provided that there is an adequate excitation mechanism.
Yet the existence of this oscillation has been a matter of much speculation. Numerous authors have claimed to have detected this motion with an amplitude of about 0.01-0.02 (see Table I of Rochester et al., 1974, for a summary of such claims). But such a retrograde wobble produces a long-period (about 460 days, for the complete theory) nutation of the rotation axis in space whose amplitude is some 450-460 times larger than the wobble amplitude, or about 4'-8" (see Toomre, 1974). Recently Capitaine and Xiao (1982) examined 14 years of Bureau International de l’Heure observations and noted a retrograde oscillation, referenced to inertial axes, of period of 434-444 days with an amplitude of only about 0.001. The 434-444 day period, compared with the predicted 460 days, is of potential interest for it implies that the factor \((\epsilon_0 - \beta) A/A_m\) in equation (25b) is in error by \((8-12)10^{-5}\). \(A_m\) would have to be in error by 4-6% or \(\beta\) would have to be in error by a factor of nearly 2. More plausible is that it is a consequence of the departure of the core flattening from hydrostatic equilibrium. Then

\[
\delta \sigma_2/\omega_0 = a_c^{-1} (\delta a_c - \delta c_c) A/A_m = (8-12)10^{-5}
\]

where \(\delta a_c\) and \(\delta c_c\) are the departures of the equatorial and polar radii of the core from its equilibrium shape. Then

\[
(\delta a_c - \delta c_c) = 200-300 \text{ m}
\]

An alternative explanation could be that the free oscillation period is significantly modified by damping of the motion, but by analogy with the Chandler wobble, this damping would have to be severe.

The nearly-diurnal free oscillation will, like the Chandler wobble, be subject to damping and may also be excited. The efficiency of any damping mechanism will depend on values of the core viscosity for viscous damping, and on the electrical conductivity of the lower mantle and core and on the magnetic field strength at the core-mantle boundary for electromagnetic damping (see, for example, Sasao et al., 1980; Sasao and Wahr, 1981). These parameters are all poorly known, making any discussions of damping and excitation mechanisms of an as yet largely unobserved oscillation, little more than speculation. Yet if such damping can ultimately be observed it would contribute to an understanding of the physical properties in the neighbourhood of the core-mantle boundary. More effective will be the study of forced oscillations with periods close to that of the free-core wobble.

4. FORCED ROTATIONAL MODES OF THE EARTH

4.1 POLAR MOTION

The dominant term in the spectrum of polar motion observations, other than the Chandler wobble, is an annual oscillation, driven by seasonal distributions in the mass of the atmosphere, oceans and ground water. Recent estimates of the annual excitation, expressed in the form

\[
\psi = \psi e^{j\sigma t} + \psi e^{-j\sigma t}
\]
are given in Table 2. The first result is from Wilson and Vicente (1980) and is based on a preliminary revision of the ILS data. The second estimate, by Merriam (1982), is based on the revised ILS data set of Yumi and Yokoyama (1980) and the differences between the two solutions are insignificant. A third result, also from Merriam (1982), is based on an 18 year BIH data set (Feissel, 1980). The significant difference between this and the ILS result reflects the general uncertainty of the retrograde component. According to Merriam, similar differences occur when the ILS and BIH data sets are compared over their common epoch and this suggests that the difference is more a consequence of the lower uncertainty of the retrograde term than of the year-to-year variability of the excitation function.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Wobble excitation</th>
<th>Author</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude (°.01)</td>
<td>Direction (Degree)</td>
</tr>
<tr>
<td>Chile 1960</td>
<td>(1) 2.12</td>
<td>114</td>
</tr>
<tr>
<td></td>
<td>(2) 2.80</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>(1) 2.56</td>
<td>109</td>
</tr>
<tr>
<td></td>
<td>(1) 2.2</td>
<td>101</td>
</tr>
<tr>
<td>Alaska 1964</td>
<td>0.72</td>
<td>201</td>
</tr>
<tr>
<td></td>
<td>0.73</td>
<td>202</td>
</tr>
<tr>
<td></td>
<td>1.11</td>
<td>203</td>
</tr>
</tbody>
</table>

Table 2 Comparison of estimates of the shift of the excitation axis due to the 1960 Chile and 1964 Alaska earthquakes. (1) is the main shock and (2) is the fore-shock of the Chile event.

Analyses of the seasonal excitation of the polar motion include the recent studies by Merriam (1982) and Wahr (1983). Both confirm the major role played by the longitudinal redistribution of atmospheric mass in exciting the annual change in the Earth's wobble (Table 2). The two studies also emphasize the importance of evaluating the ocean response to the variable surface pressure, although it does appear that the simple inverted barometer model is largely adequate. The agreement of the atmospheric excitation with the observed astronomical excitation remains poor (Figure 2), particularly in phase, and excitations other than the atmospheric mass must be investigated. Wilson and Haubrich (1976) suggested that mountain torques play an important role. Wahr (1983) concluded that both mountain torques and surface friction contribute about equally to the excitation, that neither contribution is well determined, and that each contribution represents less than about 10% of the pressure contribution. Table 2 gives the combined mountain torque and surface friction based on a general circulation model of
Figure 2 Seasonal excitation functions in polar motion expressed in the form (36) when \( \psi^+ = F_1 + jF_2 \) and \( \psi^- = G_1 + jG_2 \). The numbers correspond to the solutions given in Table 2.

The surface winds over the oceans cause currents which contribute to the relative angular momenta \( h_i \) and which produce changes in pressure on the seafloor. In so far as the direct wind effects on the annual excitation are small, these indirect effects will also be small but, according to Wahr (1983), they are not entirely negligible (Table 2). These additional terms do not lead to a major improvement between the geophysical and astronomical estimates of the excitation function.

A more important contribution to the excitation function arises from the seasonal variations in the storage of ground and ocean water. The only global estimate is still the one by Van Hylckama (1970) (Table 2) and this amounts to about 30-40% of the atmospheric pressure contribution. The variation in ground-water storage must be balanced by variations in the water stored in the oceans and atmosphere. Table 2 gives this combined result and Figure 2 illustrates the combined term and it does move the geophysical excitation into closer agreement with the observed excitation. A preliminary re-evaluation of the ground-water excitation has been made by Hinnov and Wilson (1987) who used northern hemisphere records of monthly precipitation and temperature. Their result, and their water balance term, differ significantly from the Van Hylckama result. Possibly their basic model assumptions lead to an overestimation of the function, or the southern continents play a more important role than has been assumed by Hinnov and Wilson.

Using Van Hylckama's ground-water term, the combined excitation results in good agreement with the astronomical estimate (Figure 2) and this is perhaps surprising and fortuitous in view of the variety of assumptions made, and the inadequacy of many of the geophysical data sets used, in reaching this estimate. Probably the single-greatest uncertainty arises from the ground-water term and its re-evaluation is urgently needed.
4.2 THE FORCED NUTATIONS OF THE EARTH

Astronomical observations of the principal nutations have revealed discrepancies between the observed and theoretical amplitudes and these have generally been attributed to the Earth's departures from rigidity (e.g. Jeffreys, 1948; Federov, 1963). However, these observations were generally insufficiently precise to extract useful geophysical information. Recent improvements in observational accuracies of some of the nutation terms have led to a renewed interest in examining the forced nutations of a non-rigid Earth.

The consequence of the fluid nature of the core can be evaluated using the Kelvin-Hough model of an elastic mantle and an ellipsoidally-shaped core. Consider a torque \( L_1+jL_2=L_0e^{j\sigma t} \), \( L_3=0 \), acting on such a model. The frequency \( \sigma \) is expressed relative to the body fixed axes such that it is nearly diurnal and retrograde for the long period nutations referenced to the inertial frame. That is

\[
\sigma = -\omega_0 (1-\Delta\omega/\omega_0) \tag{37a}
\]

The solution of the equations of motion contain two parts: the motion \( m \) of the rotation axis and the motion \( n = n_1+jn_2 \) of the core with respect to the mantle. The solutions are of the form

\[
m = m_0e^{j\sigma t}, \quad n = n_0e^{j\sigma t}
\]

with

\[
\begin{bmatrix}
m_0 \\
n_0
\end{bmatrix} = \begin{bmatrix}
-\sigma & -\omega_0 (1+\epsilon_c) \\
\sigma & -\omega_0 (1-\Delta\omega/\omega_0)
\end{bmatrix} \quad jL_0A_c/\omega_0\Delta
\]

(37b)

where

\[
\Delta = CA_c \omega_0^2 [\epsilon_c + (\Delta\omega/\omega_0) (-1+A_c/C)]
\]

The determinant \( \Delta \) vanishes when

\[
\Delta\omega = \omega_0 \epsilon_c/(-1 + A_c/C) \tag{38}
\]

or when the forcing function has the same frequency as the free core oscillation, or

\[
\sigma = -\omega_0 + \Delta\sigma_2 = -\omega_0 - \omega_0 \epsilon_c A/A_m \tag{39}
\]

The corresponding solution for a wholly rigid Earth is,

\[
m_0^* = -jL_0/\omega_0\Delta^r
\]

with

\[
\Delta^r = \sigma - \omega_0 (C-A)/A
\]

and the solution (37) can then be written as
where $\sigma_2$ is given by (39). The precession term in the nutation series corresponds to $\Delta \omega = 0$ and the Earth responds as if it were wholly rigid. For the principal nutation $\Delta \omega / \omega = -1/6800$ and the ratio $m_0 / m_0^r$ is reduced to about 0.993 by the fluid core. As $\Delta \omega$ approaches $\sigma_2$ the ratio departs significantly from unity. At frequencies far away from the resonance this ratio approaches the value $(1 - A_2 / A_m)$ and here the core does not partake in the motion (Figure 3).

Figure 3  Ratio of rotational response of a rigid mantle and ellipsoidal fluid core to that of a rigid planet. The frequencies of some of the principal nutations are shown as functions of the Doodson number.

The effect of the Earth's elasticity is to modify both the inertia tensor of the mantle and the core-mantle boundary. Sasao et al. (1980) give a solution analogous to (40) as

$$ m_0 = \left[ 1 - \frac{k_2 \Delta \omega}{k_0 \omega_0} \right] m_0^r - \frac{A_c}{A_m} \frac{\Delta \omega}{\Delta \sigma_2} \frac{\Delta \omega}{\omega_0} \left[ 1 - \frac{\chi}{\epsilon} \left( 1 - \frac{\Delta \omega}{\omega_0} \right) - \frac{k_2 \Delta \omega}{k_0 \omega_0} \right] m_0^r \quad (41a) $$

$$ n_0 = -\frac{A}{A_m} \frac{\Delta \omega}{\Delta \sigma_2} \frac{\Delta \omega}{\omega_0} \left[ 1 - \frac{\chi}{\epsilon} \left( 1 - \frac{\Delta \omega}{\omega_0} \right) - \frac{k_2 \Delta \omega}{k_0 \omega_0} \right] m_0^r \quad (41b) $$
where \( \epsilon \) is the dynamical flattening of the Earth, \( \chi \) is a function of the Earth's deformation, \( k_2 \) is the static Love number and \( k_0 \) is defined by (11). \( \Delta \sigma_2 \) now corresponds to the free oscillation frequency of the deformable Earth, that is, from (25b)

\[
\Delta \sigma_2 = \frac{A}{A_m} (\epsilon \chi - \beta) \omega_0
\]

(42)

The parameter \( \chi \) in (41) is of the order \( 2 \times 10^{-3} \) and values based on different Earth models vary by about 5% (Sasao et al., 1980). Now, for the principal 18.6 year nutation, \( m_0/m_0^r = 0.996 - 0.997 \) and the effect of the mantle elasticity is opposite to the core effect.

Complete discussions of the nutations of a deformable Earth are given by Sasao et al. (1980) and Wahr (1981a,c) and differences for the two theories are less than 0.001 in amplitude. Table 3 summarizes the main results. The principal departures from the rigid Earth model occur at 18.6 years and 6 months for which the discrepancies are about 0.002 - 0.003. Not included in these theories are the effects of the ocean tides and Wahr and Sasao (1981) have shown that these can be important, about 0.001 - 0.002 for the 18.6 year nutation and about 0.0005 - 0.001 for the semi-annual nutation. It is also important to recognize that this oceanic contribution is likely to lead to the observed nutations being out of phase with the predicted values for the solid Earth.

McCarthy et al. (1980) have summarized, without critical comment, some published observations for the principal nutation terms. Estimates of the 18.6 year nutation range from 9°196 to 9°214 for the obliquity and from -6°819 to -6°858 for the longitude. More recently, Capitaine and Xiao (1982) examined 18 years of astrolabe observations of the Bureau International de l'Heure (the \( z \)-term in latitude and the \( w \)-term in universal time) (Table 4). For the 18.6 year nutation their second result in Table 4 is to be preferred and these values are in satisfactory agreement with estimates by Wako and Yokoyama and McCarthy (quoted in McCarthy et al., 1980). Overall these results are in agreement with the predicted amplitudes of Wahr and Sasao et al. but the observations are inadequate to distinguish between the theoretical models. Annual and semi-annual nutations are difficult to observe with precision because of systematic errors in the observations with a seasonal character and because of the need to precisely model the seasonal changes in polar motion and length-of-day. Estimates for the semi-annual nutation have been obtained by Capitaine (1980) and E.P. Federov (quoted in McCarthy et al. 1980) but the agreement in the obliquity is not satisfactory (see Table 4). Nor do the observations for this term agree well with the realistic model predictions. Likewise, estimates for the fortnightly nutations (Table 4) differ from one solution to another, although the Capitaine and Xiao (1982) values agree well with the model.

Recent results obtained by long baseline radio interferometry hold greater promise for improved nutation observations. Herring et al. (1986) analysed four and a half years of observations of a number of baselines to estimate the nutations with periods of 1 year and shorter (Table 4) (in addition to the terms tabulated here, these authors also observed the nutations at 122; 32, 28 and 9 days) (see also Herring, 1988). Agreement
<table>
<thead>
<tr>
<th>Theory/Model</th>
<th>18.6 years</th>
<th>1 Year</th>
<th>6 Months</th>
<th>13.7 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid Earth (Kinoshita's theory)</td>
<td>9.2278</td>
<td>-6.8743</td>
<td>-0.0001</td>
<td>0.0499</td>
</tr>
<tr>
<td>Sasao et al. (Wang Earth model)</td>
<td>9.2018</td>
<td>-6.8407</td>
<td>0.0051</td>
<td>0.0565</td>
</tr>
<tr>
<td>Wahr (1966 A model)</td>
<td>9.2023</td>
<td>-6.8416</td>
<td>0.0054</td>
<td>0.0567</td>
</tr>
</tbody>
</table>

Table 3: Amplitudes (in arc seconds) of the nutations in obliquity and longitude for the principal terms in the nutation series according to the rigid Earth theory of Kinoshita (1977) and the elastic mantle-fluid core models of Sasao et al. (1980) and Wahr (1981). The nutation in longitude is defined here as \( \alpha \sin \alpha \).
with the theory is excellent at all periods except for the annual oscillation. The stated uncertainties of these nutation amplitudes are comparable or smaller than the differences predicted by the Sasao et al. (1980) and Wahr (1981c) theories, giving rise to an expectation that these observations may contribute to an improved understanding of the dynamics of the coupling of the core and mantle. In fact, Gwinn et al. (1986) interpret the above discrepancy between theory and observation at the annual frequency in terms of a departure of this boundary from the ellipticity predicted by the hydrostatic theory. The annual nutation term, referenced to the rotating frame, lies close to the frequency of the free core oscillation (Figure 3) and a small shift in this frequency caused, for example, by changing $\epsilon_c$ (equation 42) will modify the Earth’s response without it significantly affecting the other nutations. Dissipation will also change the resonance frequency but if this is assumed to be small the requisite core flattening can be computed from (49) and (50). Gwinn et al. obtain $\alpha_c - \epsilon_c = 500 \text{ m}$, similar to that deduced from the observed free oscillation period (equation 35).

5. LENGTH-OF-DAY

The non-uniform rotation of the Earth has traditionally been determined by measuring the time interval between successive transits of a star against an observer’s meridian. For the present purpose the measured quantity can be considered to be the universal time UT1. The elapsed interval is measured against a uniform reference time $R$ and the observed quantity is $\Pi = R - \text{UT1}$, the amount by which the Earth is slow after an interval $\Delta T$. The measurement of $\Pi$ therefore comprises two parts, the astronomic or geodetic part relating to the definition of the instants of transits of the celestial bodies, and the physics part relating to the establishment of a uniform time scale from frequency standards. Departures from uniform rotation are expressed in several ways, in terms of the $m_3$ defined by (1) or in terms of changes in the length-of-day. That is

$$m_3 = \frac{\omega_3 - \omega_0}{\omega_0} = - \frac{\Delta (\text{l.o.d})}{\text{l.o.d}} = - \frac{\Pi}{\text{d}t}. \quad (43)$$

Modern methods of measuring the change in length-of-day, including radio-interferometric observations of stellar sources (VLBI) and the tracking of artificial satellites and the Moon, are essentially the same in which, for example, the diurnal orientation of the VLBI baseline is monitored relative to the stellar sources.

The observational record of $\Pi$ is of variable accuracy and resolution but some observations go back to the seventeenth century (e.g. Morrison et al., 1982; McCarthy and Babcock, 1986). Until about 1955 the reference time was ephemeris time, determined from the astronomical observations themselves, and this proved to be the limiting precision of UT1 measurements. After 1955 the reference time is atomic time and the limiting factor is now the precision of the astronomical or geodetic observation but even for the past 30 years the data set is not homogeneous in accuracy and resolution.
Figure 4 illustrates the principal characteristics of the observations. The observations with respect to ephemeris time define the so-called decade fluctuations, changes in L.o.d. that persist for about 10 years and longer, rise above the noise level for at least the period from 1850 to the present. Improvements in the ephemeris time determination after about 1920 resulted in fluctuations of about 5 year duration rising above the noise level. After 1955 seasonal oscillations became clearly evident and the tidal spectrum emerged. With the present VLBI observations, considerable high-frequency changes in length-of-day are observed.

Figure 4 Schematic spectrum of the fluctuations in the length-of-day and the observational error spectra.

5.1 THE SECULAR CHANGE

The secular change in the Earth's rotation must be one of the most debated subjects in astronomy and geophysics for more than 200 years. Because of the decade-scale and longer fluctuations in the Earth's rotation, estimates of the true secular change are generally unreliable unless they are based on very long observational records but the longer the record the less reliable the data. The geophysical problem comprises two parts: one is the evaluation of the secular change resulting from the lunar and solar torques exerted on the Earth and the other is the evaluation of non-tidal contributions. The latter is generally the smaller amount, and the separation of the two remains uncertain. Causes for the non-tidal part include the Earth's adjustment to the melting of the ice sheets in Late Pleistocene time and possibly longer-period electromagnetic torques acting on the lower mantle. The subject is too vast to be discussed here and the reader is referred to the literature cited in the summaries by Lambeck (1980, 1988) (see also Rochester, 1984).
5.2 THE DECADE CHANGES

The decade changes in m³ have reached 1 part in 10⁷ over intervals of about 10 years and they are so large that their existence has been known for the best part of a century, yet their explanation remains obscure. Their magnitude is such that the atmosphere and oceans can only play a minor role in their excitation and the consensus is that the explanation must lie in differential motions between the core and mantle (Munk and MacDonald, 1960; Lambeck, 1980; Rochester, 1984). Where the disagreement occurs is the question of the mechanism(s) by which core motions are coupled to the mantle. Is it by viscous friction, by electromagnetic torques, or by the pressure of the core fluid as it flows past a non-spherical core-mantle boundary? Disagreement is not so much a matter of details about the mechanisms as one of the choice of appropriate physical parameters describing the core and lower mantle.

With a change in m³ of 10⁻⁷, as occurred from about 1870 to 1900, the torque required to act across the core-mantle boundary is about 10¹⁸ Nm. The strength of the coupling locally is defined by a tangential stress $\tau$ at the boundary and the net torque exerted on the mantle is

$$L = \int_{S} R_{B} \wedge \tau_{t} \cdot \frac{e_{r}}{dS}$$

where $e_{r}$ is the normal to the core-mantle boundary $R_{B}$. The average tangential stress at $R_{B}$ therefore has to be of the order 10⁻² Nm⁻².

Surface friction gives rise to a stress

$$\tau_{v} = \rho \eta_{*} \frac{v}{D}$$

where $\rho$ is the density, $\eta_{*}=\eta \rho^{-1}$ is the kinematic viscosity, $v$ is the velocity of fluid past the boundary, and $D=(\eta_{*} / \omega)^{2}$ is the boundary layer thickness (Hide, 1977). An estimate of $v$ is obtained from the westward drift of the magnetic field, at an average rate of about 0.2° a⁻¹, as this phenomenon is believed to be indicative of differential rotation between the core and mantle (but see Hide, 1966) so that $v=4x10^{-4}$ m s⁻¹. Hence, for coupling to be effective, the core kinematic viscosity needs to exceed 0.1 m² s⁻¹. Few geophysical observations place strong constraints on this viscosity. Observations of the damping of seismic waves, for example, determine an upper limit of about $\eta_{*}<10^{4}$ m² s⁻¹, while estimates based on the nutation observations requires that $\eta_{*}<10$ m² s⁻¹ (Toomre, 1974). Theoretical and physical estimates place much lower bounds on this parameter and Gubbins (1976) proposed that $\eta_{*}=4x10^{-7}$ m² s⁻¹ while Bukowinski and Knopoff (1976) suggested a value of about $10^{-4}$ m² s⁻¹. Based on these estimates, viscous coupling is generally believed to be unimportant.

Topographic coupling, by the flow of core-material past an irregular boundary has been proposed by Hide (1969,1977), and the mechanism is similar to the mountain torques exerted by the winds on the Earth's surface. The local stress is approximately (Hide, 1977)

$$\tau_{r} = C_{r} \rho_{w} v (H-D)$$
where $C_t$ is a drag coefficient, and $H(D)$ is the height of the irregular topography of the boundary. $C_t$ is a function of $H$ and wavelength of the bumps as well as of the magnetic field properties (Moffatt, 1978; Hassan and Eltayeb, 1982). Generally $H>>D$ and the value of the viscosity of the core is unimportant in the evaluation of $\tau_r$. Hide suggested that $C_t=1$ but Hassan and Eltayeb proposed $C_t=10^{-3}$ and with the latter value $H$ must approach 10 km for $\tau_r$ to be significant but, from other geophysical evidence and arguments, this appears excessive. For $C_t=1$ the mechanism becomes a much more plausible candidate but clearly more work is required to establish appropriate limits on this coefficient.

Electromagnetic coupling has been introduced in section 3.1.5. The time constant for this coupling $\tau_3$, defined by (33), is of the order 8-15 years and quite adequate to explain the decade-scale length-of-day changes. The local tangential stress at the core-mantle boundary

$$\tau_3 = C_e B_r B_t / \mu$$

where $C_e$ is an electromagnetic "drag coefficient" and is approximately equal to unity, $\mu$ is the magnetic permeability and is not significantly different from the value for free space, and $B_r, B_t$ are the radial and tangential components of the magnetic flux density $\mathbf{B}$ at the core-mantle boundary (Rochester 1962; Hide 1977). $B_r$ can be estimated by extrapolating the surface field down to this boundary and more problematical is the field $B_t$, induced by the relative motion of the conductive core and lower mantle in the presence of the radial field, its value being a function of the motions within the core and of the conductivity of the lower mantle. In some of the early dynamo models $B_t$ was found to be 100-200 times larger than $B_r$ but in more recent models $B_r=B_t$ (e.g. Busse 1975). With either case, most investigators agree that $B_t$ will be adequate to produce the requisite stress at the core-mantle boundary (Hide 1977; Rochester 1984).

If electromagnetic coupling does occur, some correlation between fluctuations in magnetic field parameters and the planet's rotation may be anticipated. Several attempts have been made to determine such correlations between, for example, l.o.d. and changes in the drift rates of long wavelength components of the field or the variation in the intensity of the dipole term in the field (e.g. Yukutake 1973). Such correlations are particularly important for studying the conductivity structure of the lower mantle. The time constant $\tau_1$ for a change in the magnetic field to diffuse through the lower mantle is given by equation (33a) so that a delay is expected to exist between the Earth's rotational response and any magnetic field changes at the surface, by an amount that is a function of the electrical conductivity. Evidence for these correlations is limited. First, the extrapolation of the field $B_r$ to the core-mantle boundary is poorly constrained for the higher wavenumber components. Second, the component $B_t$ is reduced to zero at the surface of the Earth because of the very low conductivity of the upper mantle and estimates of the magnitudes of these fluctuations are little more than speculations. Thus an absence of correlation cannot be used to infer the inadequacy of electromagnetic core-mantle coupling.

The coupling models examined so far generally consider only long wavelength terms in the magnetic field, although the analysis by Stix (1982)
includes harmonics up to degree 12. Higher harmonics may be very effective in coupling core and mantle motions, particularly when the conductivity gradients are steep (Rochester, 1984), but they are attenuated rapidly as the Earth's surface is reached and at some wavelength, they cease to penetrate through the mantle so that they cannot be observed. If these higher harmonics are effective in producing high frequency coupling this should show up in the length-of-day spectrum although the latter contains little unexplained power at periods less than about 4-5 years and it does not appear that significant coupling occurs at these shorter periods.

5.3 METEOROLOGICAL EXCITATION

The principal contribution to the seasonal changes in the earth's rotation comes from the zonal circulation in the atmosphere and the evaluation of the excitation function is straightforward once this circulation is known over the globe and up to altitudes above the tropopause. From (5)

$$\psi_3 = \psi_3^{\text{matter}} + \psi_3^{\text{motion}} = -\Delta I_{33}/C - h_3/\omega_0C$$

and the complete evaluation requires a knowledge of both the mass redistribution ($\Delta I_{33}$) and the winds ($h_3$) in the atmosphere. Evaluation of the two terms has shown however that the relative motion term is up to an order of magnitude greater than the matter term and the principal task of evaluating $\psi_3$ is the measurement of the zonal winds.

The astronomically observed annual and semi-annual oscillations in l.o.d are well explained by the combination of wind excitation and tides (see below) (Lambeck and Hopgood, 1982) but nevertheless several limitations of the current wind data sets remain. These include the general lack of global wind data above 100 mbars for, as shown by Lambeck and Cazenave (1973) and Rosen and Salstein (1985), winds up to stratospheric heights are important. A second limitation is the paucity of data for the southern hemisphere. Apart from the annual and semi-annual terms, other oscillations at periods near 120 days, from 40 to 50 days, and shorter have been noted in the length-of-day spectrum, as have longer periods of 2-3 year duration. Most of these fluctuations are of meteorological origin (Rosen and Salstein, 1985).

Major efforts are underway at several centres to compute systematically the meteorological excitation functions, both $\psi$ (motion) and $\psi$ (matter), on a daily basis (e.g. Whysall et al., 1985; Naito and Yokoyama, 1985; Salstein and Rosen, 1985) and it appears that this excitation can now be removed with some confidence that the residual excitation has geophysical meaning. Unfortunately, within the present levels of accuracy of both $\psi_3$ and $m_3$, not much remains at these sub-seasonal periods other than the tidal signals (see Figure 5). The importance of being able to "correct" the astronomical data for the meteorological "noise" cannot be over emphasized for not only does it mask possible high-frequency solid-earth signals, it also contaminates the Earth's response to the lunar tide signals (see below). The interpretation of the decade changes in l.o.d is also complicated by the meteorological excitation. It is not suggested that the atmosphere makes a major contribution to the excitation but it may mask essential characteristics.
of these changes. For example, do these changes in length-of-day occur rapidly, driven by short-duration impulses in electromagnetic torques at the core-mantle boundary? Or do the changes occur more gradually, over periods of a few years. Because atmospheric contributions to the total excitation are significant at periods up to a few years this question cannot be answered with certainty unless the meteorological excitations are known.

5.4 TIDAL SIGNALS IN THE EARTH'S ROTATION

Jeffreys, in 1928, was the first to point out that the conservation of angular momentum of the earth implied that there must be periodic changes in the

---

**Figure 5** Power spectra of (a) $m_3$, (b) the zonal wind excitation $\psi_3$, and (c) $m_3-\psi_3$ less tidal signal. All spectra are based on the time interval from 1958 to 1980. The error spectra are indicated by the dashed lines (from Lambeck and Hopgood, 1982).
length-of-day. However, the semi-annual term in l.o.d was not observed until about 1953 and its early interpretation was complicated by errors in the FK3 star catalogue then in use, and by the recognition that seasonal factors, other than tides, could contribute. Observations of the monthly and fortnightly tidal terms were first reported by Markowitz in 1955, from an analysis of photo-zenith-tube observations. With the now available long series of length-of-day referenced to atomic time, the tidal terms rise clearly above the background measurement noise (e.g. Merriam, 1982; Yoder et al., 1981).

Detailed theoretical calculations of the tidal perturbations were carried out by Woolard in 1959 for an elastic Earth. If a fluid spherical core is introduced with no coupling at the core-mantle boundary then the core will not take part in the rotation and the amplitudes for an elastic Earth will be reduced by \((A-A_m)/A_m\) or about 10%. The effect of the ocean tide is to increase the effective flattening of the Earth, assuming that this tide follows an equilibrium theory, and this increases the length-of-day changes, also by about 10%, so that the effective "whole Earth" Love number is actually close to that of an elastic Earth.

5.4.1 Solid Tides

A qualitative estimate of the changes induced in the Earth's rotation by the tidal deformation of the Earth follows readily from the Love number description of this deformation. The tide-raising potential of degree 2 and order \(m\) at \(r=R\) can be written as

\[
V_2 = \sum_{r} V^{(r)} = G_0 \left[ \frac{R}{a_c} \right]^2 \sum_{m=0}^{\infty} B_{2m}^{(r)} P_{2m}(\sin \varphi) \cos(\sigma_{2m}^{(r)} t + \lambda + \beta_{2m}^{(r)})
\]

(44)

the frequencies, phase angles and amplitudes of these components. \(a_c\) and \(m_c\) are the semi-major axis of the lunar orbit and the lunar mass respectively. The corresponding rotational excitation functions are, neglecting the time derivative of the inertia tensor in the equations,

\[
\psi_1 = m_c \left[ \frac{R}{a_c} \right]^3 \left( \frac{a}{c} \right)^2 k_2 B_{21}^{(r)} \left[ \cos(\sigma_{21}^{(r)} t + \beta_{21}^{(r)}) \right]
\]

(45a)

\[
\psi_2 = m_c \left[ \frac{R}{a_c} \right]^3 \left( \frac{a}{c} \right)^2 k_2 B_{20}^{(r)} \cos(\sigma_{20}^{(r)} t + \beta_{20}^{(r)})
\]

(45b)

For \(m=0\), the tidal potential has a zonal geometry and its time dependence is of long period. These tides contribute only to \(\psi_3\) and \(m_3\). For \(m=1\) the periods group around 24 hours and the polar motion is subjected to small nearly-diurnal oscillations. There are no first-order contributions from the semi-diurnal \((m=2)\) tides because the first order equations of rotational motion are independent of the inertia elements \(I_{12}\) and \((I_{11}-I_{12})\). The diurnal tides contribute significantly to \(\psi\) but the corresponding motion of the rotation axis has an amplitude that is approximately \(\sigma_0/\sigma^{(r)}\) or 1/435 of
ψ and the diurnal tidal effect on the Earth's rotation becomes significant only at the very highest levels of precision and resolution of observations of the Earth's rotation.

The zonal tides play a considerably more important role. Lunar tides cause perturbations in length-of-day near 14 and 27 days and solar tides cause perturbations near 6 months and 12 months. Longer period perturbations occur near 8.8 years and 18.6 years, although their amplitudes are small when compared with the observed decade changes.

5.4.2 Core effect

In analogy with the Earth's rotational response to the lunar and solar torques, the tidal deformations of the fluid core may be decoupled from the mantle deformations. As noted by Merriam (1980) (see also Wahr et al., 1981; Yoder et al., 1981), this suggests that, instead of (5b) the rotational excitation function ψ3 should be written as

\[ ψ_3(\text{matter}) = -\frac{ΔI_{33}^{m}}{C_m} \]  

where \( ΔI_{33}^{m} \) refers to the change in the inertia tensor of the mantle only. The evaluation of these elements requires a solution of the equations of deformation for a planet with an ellipsoidally shaped fluid core that is acted upon by a body force potential of degree 2. The solution is (Wahr et al., 1981)

\[ ψ_3 = -\frac{ΔI_{33}^{m}}{C_m} = -\frac{ΔI_{33}}{C} \left(1-C_C γ/C_X\right)/(1-C_C/C) \]  

where \(-ΔI_{33}/C\) corresponds to the excitation function for the elastic Earth model, \( C_C \) in the polar moment of inertia of the core, \( γ \) is the non-dimensional deformation constant and \( \chi=(k_2/k_0)(C-A)/A \). This indicates that the tidal response can then be written in the form (59) but with the elastic Love number replaced by the parameter

\[ κ = k_2(1 - C_C γ/C_X)/(1-C_C/C) \]  

The parameters required to evaluate this equation are given by Sasao et al. (1980) for three different Earth models and \( κ \) ranges between about 0.266 and 0.268. The effect of the fluid core is therefore to reduce the elastic tidal response by about 10-11%.

If some degree of core-mantle coupling occurs, the effective Love number will lie between the above value for \( κ \) and the elastic value \( k_2 \). A lag angle will also be introduced into the response but this is unlikely to reach a magnitude where it can be observed (see Yoder et al., 1981; Wahr et al., 1981).

5.4.3 Ocean tides

The ocean tide contribution to the excitation can be evaluated in a manner that is similar to that used to examine the oceanic pole tide effects on the Earth's rotation. This contribution comprises two parts; the direct
deformation of the ocean which, if the tide follows an equilibrium theory, will result in an increase in the tidal elements of the inertia tensor, and the indirect contribution, of opposite sign, arising from the Earth's deformation under the ocean tide load. The excitation function follows by expanding the ocean tide into spherical harmonics and the result is (Lambeck 1980)

\[
\psi_1 = \frac{4\pi}{3} \rho_w \frac{R_e^4}{(C-A)} (1+k_2) D_{st}^z(\nu) \left[ \cos(\sigma(\nu)t-\tau_{st}(\nu)) - \sin(\sigma(\nu)t+\tau_{st}(\nu)) \right]
\]

\[
\psi_2 = \frac{8\pi}{15} \rho_w \frac{R_e^4}{C} (1+k_2) D_{st}^r(\nu) \cos(\sigma(\nu)t+\tau_{20}(\nu))
\]

where the \(D_{st}^z(\nu)\) and \(\tau_{st}(\nu)\) are parameters that define the tide with respect to the seafloor which itself is deformable. This latter response is summarized by the load Love numbers \(k_2\). It is important to note that the spectrum of the tidal perturbations in the planet's rotation will be more complex than for the elastic Earth model with or without a fluid core. The semi-diurnal ocean tide expansion, for example, contains terms harmonic in degree and order 2,0 and 2,1. Thus, while there are no solid Earth contributions from this tide-potential (ignoring the second order terms in the equations of motion), there will be small semi-diurnal terms in both the polar motion and the length-of-day. Likewise, the long-period zonal tides contain harmonics in degree and order 2,1 and it could be anticipated that the polar motion contains small oscillations with the same periods as these long period tides. These excitation functions consider only the term \(\psi_1(\text{matter})\) and ignore possible contributions from \(\psi_1(\text{motion})\) arising from tidal currents, and this neglect warrants further investigation.

Consider the semi-diurnal M2 tide. The amplitude \(D+\) of the 2,1 component is about 2.5 cm so that \(V_1=2.8\times10^{-7}\) and \(|m_1|=3\times10^{-10}\) and the semi-diurnal oscillation in polar motion is entirely negligible. The 2,0 coefficient in the M2 tide has an amplitude of about 1 cm so that \(V_3=1.2\times10^{-10}\) and the change in length-of-day is about 0.02 msec. This is also below the present observational noise level. Likewise, the 2,0 coefficients in the diurnal tides produce negligible diurnal oscillations in the length-of-day.

The (2,1) harmonics in the diurnal ocean tide introduce perturbations in \(m\) that are of similar magnitude, or about 10% of the elastic tide effect, and these also are wholly negligible. But this is not true for the nutations in space for while the proportional change is the same, the amplitudes themselves are much larger: if the elastic deformation modifies the nutation amplitudes by about 30% then the ocean effect is about 3% of the rigid Earth nutation and there will be a small lag in the motion of the rotation axis with respect to the direct lunar and solar potential.

Whether or not the Mm and Mf ocean tides follow an equilibrium theory remains a matter of debate. Here it will be assumed that such a model is valid and Table 5 gives the corresponding perturbations in rotation. The largest perturbations in polar motion occur for the Mf tide although the amplitudes are small, with \(|m_3|=2\times10^{-9}\) or less. The perturbations in \(m_3\) are considerably more significant, about 10% of the corresponding solid tides.
THE EARTH'S VARIABLE ROTATION: SOME GEOPHYSICAL CAUSES

<table>
<thead>
<tr>
<th>Darwin notation</th>
<th>Doodson number</th>
<th>Period (sol. days)</th>
<th>( m_3 ) (Elastic Earth) ( \times 10^{-9} )</th>
<th>( m_3 ) (Core+Ocean) ( \times 10^{-9} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>055.565</td>
<td>6817</td>
<td>1.654</td>
<td>1.730</td>
<td></td>
</tr>
<tr>
<td>055.575</td>
<td>3409</td>
<td>-0.016</td>
<td>-0.017</td>
<td></td>
</tr>
<tr>
<td>Sa</td>
<td>056.554</td>
<td>366.3</td>
<td>-0.292</td>
<td>-0.305</td>
</tr>
<tr>
<td>SSa</td>
<td>056.556</td>
<td>366.2</td>
<td>-0.015</td>
<td>-0.016</td>
</tr>
<tr>
<td>MSm</td>
<td>057.555</td>
<td>183.1</td>
<td>-1.840</td>
<td>-1.925</td>
</tr>
<tr>
<td>Mm</td>
<td>057.565</td>
<td>178.3</td>
<td>0.045</td>
<td>0.047</td>
</tr>
<tr>
<td>058.554</td>
<td>122.0</td>
<td>-0.107</td>
<td>-0.112</td>
<td></td>
</tr>
<tr>
<td>Msf</td>
<td>063.655</td>
<td>31.9</td>
<td>-0.399</td>
<td>-0.417</td>
</tr>
<tr>
<td>065.455</td>
<td>27.7</td>
<td>-2.084</td>
<td>-2.180</td>
<td></td>
</tr>
<tr>
<td>Mf</td>
<td>073.555</td>
<td>14.8</td>
<td>-0.346</td>
<td>0.362</td>
</tr>
<tr>
<td>075.555</td>
<td>13.7</td>
<td>-3.959</td>
<td>-4.141</td>
<td></td>
</tr>
<tr>
<td>075.565</td>
<td>13.6</td>
<td>-1.642</td>
<td>-1.718</td>
<td></td>
</tr>
<tr>
<td>085.455</td>
<td>9.1</td>
<td>-0.758</td>
<td>-0.793</td>
<td></td>
</tr>
</tbody>
</table>

Table 5 Amplitudes of perturbations in \( m_3 \) caused by the long period zonal tides. The fourth column corresponds to an elastic Earth with \( k_2 = 0.30 \) while the fifth column corresponds to an elastic mantle, fluid core and equilibrium ocean.

The tidal response of an elastic Earth with an equilibrium ocean tide is described by the effective Love number

\[
\kappa = k_2 (1 + C \psi_3^{ocean} / k_2 \Delta I_{33})
\]

and equals about 1.125 \( k_2 \) - 1.129 \( k_2 \) for the above model (Agnew and Farrell, 1978; Merriam, 1980; Lambeck, 1980). If a non-equilibrium tide is used, \( \kappa \) will include an imaginary part, reflecting the lag of the combined elastic-ocean tide with respect to the tide raising potential. The total tidal excitation function is, with (47),

\[
\psi = -k_2 \frac{\Delta I_{33}}{C} \left[ x_c + C \psi_3^{ocean} / k_2 \Delta I_{33} \right] = -\kappa \Delta I_{33} / C \tag{50a}
\]

where

\[
x_c = \frac{[1 - C_C \gamma / C_C]}{(1 - C_C / C)} \tag{50b}
\]

and for the above models \( \kappa = 1.03 \) - 1.04 \( k_2 \).
Numerous analysis of length-of-day observations for the tidal Love number $k$ have been made, but no consistent result appears. For example, Guinot (1974) and Lambeck and Cazenave (1974) found that $k_{Mf} > k_{Mm}$ although both estimates were found to be quite variable from data set to data set. Others, for example Yoder et al. (1981), found that $k_{Mf} = k_{Mm}$. Capitaine and Guinot (1985) noted that $k_{Mf}$ exhibits considerable variability with time whereas $k_{Mm}$ appears to be relatively stable. This irregular type of behaviour led Lambeck and Cazenave to suggest that there is an increase in the power of the meteorological excitation about these tidal frequencies and the zonal wind excitation functions of Rosen and Salstein (1983) confirm this. Thus the length-of-day observations should first be corrected for this atmospheric excitation before Love numbers are estimated. Merriam (1984) has noted that some of the difference between the $Mm$ and $Mf$ results is also a consequence of the use of Woolard's nutation series instead of the Wahr (1981b) theory, although the Capitaine and Guinot results are based on this latter theory and the problem remains.

Merriam (1984) appears to be the only one who has estimated the Love numbers by first correcting the length-of-day data for the atmospheric excitation and he found that, within observational errors, $k_{Mf}$ equals $k_{Mm}$ and that the statistics of the atmospherically corrected results are better than those of the uncorrected length-of-day data set (see Table 6). With (50), the equivalent Love number for a static Earth model without oceans is about 0.309 and this is marginally greater than the value predicted from seismological data by an amount that is consistent with the frequency dependent $Q$ law with $\gamma = 0.2$. The uncertainties, however, are sufficiently large to permit a wide range of $\gamma$ values, from 0 to more than 0.3. It should be noted that the load Love numbers will also exhibit dispersive behaviour so that the ocean loading term is frequency dependent and for detailed studies Merriam's formulation is preferred. Longer wind data sets are now available and a recalculation is worthwhile.

<table>
<thead>
<tr>
<th>BIH</th>
<th>Phase (days)</th>
<th>Wind corrected</th>
<th>BIH</th>
<th>Phase (days)</th>
<th>Static Love number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{Mf}$</td>
<td>0.331±0.023</td>
<td>+0.15</td>
<td>$k_{Mf}$</td>
<td>0.317±0.013</td>
<td>0.304</td>
</tr>
<tr>
<td>$k_{Mm}$</td>
<td>0.364±0.016</td>
<td>-0.09</td>
<td>$k_{Mm}$</td>
<td>0.327±0.011</td>
<td>0.314</td>
</tr>
<tr>
<td>Mean</td>
<td>0.322</td>
<td></td>
<td>Mean</td>
<td>0.322</td>
<td>0.309</td>
</tr>
</tbody>
</table>

Table 6  Zonal Love number $k$ at the $Mf$ and $Mm$ frequencies before and after the removal of the atmospheric excitation from the BIH data from 1978-1982. A positive phase means that the observed signal leads the nominal tide signal. (From Merriam 1984). The static Love number is $k$ corrected for the equilibrium ocean tide (and loading) and the extent to which the core is not coupled to the motion of the mantle.
6. CONCLUSIONS

The very nature of the Earth's rotation requires observational records that extend over many years and even centuries. In consequence, the new measurement procedures based on space-age technologies have not yet made a major impact on the geophysical discussion of those motions. This is in part because the technologies themselves have evolved rapidly and homogeneous data sets do not yet exist for even a few years. Thus the Doppler satellite observations of the Earth's irregular rotation, which represented a major improvement over the classical astrometric observations, have themselves been supplanted by the laser range observations of satellites and the long baseline interferometric observations of stellar radio sources. Yet the geodetic developments of the past two decades have now reached the point where one can see geophysical signatures arising out of the noise from only relatively short series of observations and one example of this is the VLBI observations of the Earth's short-period nutations.

It would be hazardous to predict where our understanding of the deformations of the Earth will be when several decades of precise rotation observations are available. New responses to known driving forces are likely to be discovered and as yet unknown mechanisms will be postulated. One reason why such a prediction is hazardous is that the levels of precision of the observations are now such that they are much contaminated by meteorological excitations and what will be required in order to exploit fully the new geodetic measurements is a parallel program of the appropriate global atmospheric-oceanic-hydrologic parameters. Another reason why the prediction is hazardous is that there will be progress in complementary areas of geophysics as well. Advances in seismic tomography, for example, will undoubtedly lead to an improved understanding of the core-mantle boundary and to improved insight into core-mantle coupling processes.

REFERENCES


