GFF volume 128 (2006), pp. 349–351. Debate

Hyperbolic tangents are no substitute for simple classical physics

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The Earth is not shaped by mathematical functions but by physical processes. The mathematics used to describe the observational consequences of such processes must therefore be consistent with the physics if the outcomes are to provide useful predictions or understanding. To use a particular mathematical function to describe distinctly different processes, irrespective of the underlying physics, therefore runs the risk of reaching unfounded conclusions. This is the risk that Påsse & Andersson (2005) runs when they uses their hyperbolic tangent functions to describe isostatic rebound and eustatic sea level, as well as the fluctuations in the radiocarbon time scale, rather than use the more conventional approach of describing the physics of earth deformation and of ice sheet evolution. Or, for that matter, than use the experimentally derived calibration of the radiocarbon time scale.

Hyperbolic functions are combinations of exponential functions, and for linear materials the stress relaxation can usually be defined as an exponential decay. Thus it is not surprising that combinations of such functions can approximate rebound phenomenon. But describing an observation and understanding the underlying processes are two different things and the hyperbolic function description will not shed much light on the other.

Påsse & Andersson's justification for the use of the hyperbolic functions appears to be that 'glacio-isostatic movement starts slowly, reaches a maximal rate and after that follows a declining course', a statement that he attributes to Andrews (1970). I have no argument with that. But this statement contributes two pieces of information: it says something about the rates of melting of the ice sheet and about the Earth's rheological response. Fitting a hyperbolic function to such an observation provides a parameter (or parameters) that include both pieces of information but that cannot be separated because there is no physics to do so. If the decay of the ice sheet was not initially slow, as the above statement implies, but instantaneous, then a simple exponential function may suffice (assuming a single relaxation time for the earth). But again, the constant would be a function of the amount of ice that disappeared and of the rheology. Thus if different hyperbolic functions are found at different localities, does this mean that the description of the ice load differs in the two areas, that there is a depth dependence of viscosity resulting in several relaxation constants whose relative importance is a function of ice-sheet dimension, or that the rheological response is regionally variable? There is no way of knowing and important information is lost.

One feature of hyperbolic functions is that they are continuous. Thus they do not represent well processes that are characterised by abrupt change and this can be seen in Fig. 7 where the carefully documented evidence of Svensson (1989, 1991) for a very rapid or near-instantaneous fall in water level is stretched out over a period that is longer than supported by the field data. Thus it is probably pre-ordained that he reaches the conclusion that there was no elevated Baltic Ice Lake before ~ 11 500 BP (c.f. Björck 1995). Likewise, the use of these functions to describe the eustatic change also precludes there from being very rapid changes in the sea-level response.

An experiment simple in principle, if time-consuming in execution, illustrates that different conclusions are reached if the description of the observations follows physical processes instead. Consider all sea-level data for Scandinavia and exclude from it data from within the Baltic for the period up to the start of the Litorina. Consider separately the data beyond the former ice margins where the isostatic signals are small and where it is possible to estimate the change in total ice volume with only relatively simple ice models (e.g. Lambeck et al. 2002). Then ignoring, for descriptive convenience only, gravitational and isostatic contributions from the more distant ice sheets, the crustal rebound can be estimated as the difference between the observed local sea-level change and the eustatic component from the more global data. The magnitude and time-behaviour of this rebound function will be dependent on the ice history back to the Last Glacial Maximum and earlier as well as on the rheology of the mantle. Neither are perfectly known but both can be improved from an inversion of field data if there are physical relations that permit both the ice and earth models to be described by a number of parameters such as the effective elastic thickness of the lithosphere, effective viscosities for the mantle, and basal stress at the rock-ice interface (e.g. Lambeck et al. 1998a). The result of the inversion is a model for the ice sheet and for the mantle that will usually provide a good description of the observations. And it will also have a physical basis so that it can predict other manifestations of the rebound, such as direct measurements of crustal displacement (Milne et al. 2001), lake tilting (Lambeck et al. 1998b) or the Earth's rotation. Equally important, the resulting parameters can be compared with other estimates of mantle rheology that are independent of the rebound analysis (Cadek & Fleitout 2003).

If we return to the Baltic data that was excluded from the analysis, then we can predict the sea levels at the Baltic sites and compare it with the observed values. Any differences, observational and model errors apart, will then indicate whether the Baltic remained at sea level throughout the late- and postglacial interval. If systematic differences are found the Baltic lake-level observations can be reduced to coeval sea level and the entire calculation can be iterated to ensure that convergence is achieved. This is the simple calculation referred to and was initially done with a relatively small dataset (Lambeck 1999). The result of the first iteration of a more complete solution is illustrated in Fig. 1. The observational database includes over 3000 observations across Scandinavia of which some 1200 are from within the Baltic Basin. In this first pass solution no filtering of data has been applied and the result is not perfect. A cursory examination of the results point, for example, to some regional discrepancies that reflect a need to improve the ice model within the Baltic Basin and this should be examined in the next iteration. But this outcome is consistent with the widely accepted understanding of the history of the Baltic (e.g. Björck 1995): an elevated lake level from the time the Baltic first became ice free until ~ 11 500 years BP, possibly with an early drainage event at ~ 13 000 years

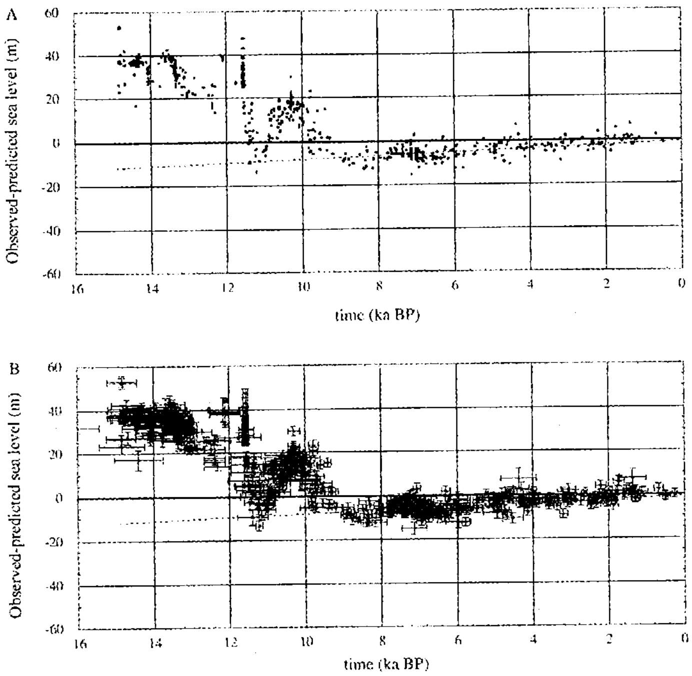

Fig. 1. First iteration solution for the Baltic Lake-Level Function, representing the elevation of the Baltic Sea relative to coeval sea level. In **A** the solution is shown without error bars and in **B** error bars are included. Some of the scatter shows regional patterns that can be attributed to ice model limitations within the Baltic basin where the first iteration solution is weak because this data has not been used in the inversion for ice-sheet parameters. The negative trend during the Litorina, and possibly extending to the Yoldia stage points to long-wavelengths limitations in the first-iteration solution, in particular to the solution for the lower-mantle viscosity which is poorly constrained by the Scandinavian data alone (Lambeck et al. 1998a).

BP, followed by a brief period when the Baltic was at sea level and then the familiar Ancylus transgression and regression. At the same time, as earlier published iterations of the Scandinavian rebound have shown, the analysis has lead to new insights into the evolution of the ice sheet from the time of the LGM as well as constraints on mantle rheology on time scales of 10^3 to 10^4 years. It demonstrates that going of on hyperbolic tangents can not only lead to odd conclusions but also to failure to extract useful information from the field data.

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Never forget the highest coastline! – Reply to "Hyperbolic tangents are not substitute for simple classical physics"

TORE PÅSSE

In his debate article Lambeck (2006) concludes that hyperbolic tangents are no substitute for simple classical physics in shore-level modelling. His conclusion is a critique of an article by Påsse & Andersson (2005) with the title "Shore-level displacement in Fennoscandia calculated from empirical data". Lambeck´s conclusion is quite right but the aim with our article was not to present any geophysical explanation of the shore-level displacement. Geophysical assumptions and hypothesis we leave to scientists with appropriate knowledge. The intention with our model was exclusively to describe shore-level displacement from empirical data in a detailed manner and to make palaeogeographic maps by combining the calculated shore-level information with a Digital Elevation Model. I will point out that field data shows that we have succeeded in that intention.

Regarding the hyperbolic tangents Lambeck (2006) actually admit "that combinations of such functions can approximate rebound phenomenon". It is obvious that his critique is not directly to discredit our method. Instead, his intent is to defend the traditional story about the existence of the Baltic Ice Lake. I need to make clear that that our article does not completely exclude the existence of the Baltic Ice Lake. In our article, we conclude that the Baltic Ice Lake only can have existed for 100 or 200 years and that the damming effect therefore must have been just a few meters. However, before I focus on the Baltic Ice Lake I will comment on our usage of hyperbolic tangents.

Lambeck (2006) questioning our expression for calibration the radiocarbon time scale. There is essentially no difference in our calibration curve and that of Stuiver and Reimer (1993). I think