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DETERMINATION OF EARTH AND OCEAN TIDES FROM THE ANALYSIS OF SATELLITE ORBITS

1. Solid Earth Tides

The elastic deformation of the Earth due to the variable lunar and solar attraction has been reviewed most recently by SLICHTER (1972). Observations of these deformations provide estimates of the Earth's mean elastic parameters. If the potential of the attracting force $U_n$ is harmonic in degree $n$, the elastic response is also assumed to be harmonic in degree $n$, and in the classical definition of LOVE (1909), (see also JEFFREYS 1962) the additional potential resulting from the deformation is defined at the Earth's surface $R$, as

$$\Delta U_n = k_n U_n(R)$$

or, at a point $r$ exterior to the Earth, as

$$\Delta U_n(r) = k_n \left( \frac{R}{r} \right)^{n+1} U_n(R) = k_n \left( \frac{R}{r} \right)^{2n+1} U_n(r)$$

The principal tidal terms occur for degree 2. The harmonics of zero order give the zonal tides, the first order harmonics the diurnal tides and the second order harmonics the semi-diurnal tides. Similar definitions define the actual deformation at the Earth's surface. Thus the radial deformation at the surface is defined as $h_n U_n(R)/g$ (g is gravity at the surface) and the horizontal deformations are in longitude ($\lambda$) $k_n \frac{\delta h_n}{\delta \lambda} U_n(R)$, and in latitude ($\phi$) $\frac{\delta h_n}{\delta \phi} U_n(R)$. The Love numbers $h_n, k_n, \delta h_n$ are integral measures of the Earth's elastic properties and relate in a complex manner to such parameters as the density, bulk modulus and rigidity variations throughout the Earth. They present most satisfactory transfer parameters between complex theory on the one hand and refined measurements on the other hand, although their interpretation is not always free from ambiguity.

In their classical definition, the Love numbers define the response of a radially symmetric, perfectly elastic, Earth to the perturbing potentials. This theoretical response is now best determined from seismology where the elastic parameters can be measured directly as a function of depth. These calculations have been most recently performed by FARRELL (1972). This study, as well as earlier ones, showed that the Love numbers are not very sensitive to the choice of mantle model; an oceanic type upper mantle giving almost identical results as a continental type upper mantle. Thus, if the theoretical response corresponded to reality, there would be little value in observing the solid tides as they provide only an insensitive global measure of the elasticity of the Earth. Before the development of seismology, however, solid tide observations played an important role in establishing the existence of the dense and liquid core.
The theoretical response is strongly modified or influenced by the fluid parts of the Earth, and to a lesser extent by the Earth's aelastic properties. A modification of some interest is the possible resonance effect due to inertial coupling between the elastic mantle and fluid core as propounded by the theories of JEFFREYS & VICENTE (1957) and which predict, for some of the diurnal tides, a rapid change in the value of $k_2$ with frequency. Solid tide observations have until now, not been very successful in distinguishing between the variously proposed models mainly because the oceans perturb the tide observations (SLICHTER 1972; BLUM et al 1973). In any case, it would appear that these resonances may tell us more on how to solve an interesting mathematical problem rather than tell us about the physics of the coupling mechanism itself.

The Earth is not a purely elastic body, for if it were, it would still be vibrating under the combined effect of all the earthquakes since its origin. Energy is therefore dissipated, and in the case of the tidal problem this dissipation results in a slight delay in the response to the attracting potential. Observation of this lag is of greater intrinsic value than the Love numbers themselves as it provides a measure of the Earth's global imperfections in elasticity at the tidal frequencies and provides a key parameter in the understanding of the evolution of the Earth-Moon system. Observations of this lag have until now not been particularly conclusive also because the measurements are perturbed by the ocean tides.

2. Ocean Tides

Ocean tides have been reviewed recently by HENDERSHOTT & MUNK (1970) and HENDERSHOTT (1973). Long records of ocean tides exist along many of the world's coastlines and these are extremely valuable for predicting the tides locally. But such observations are very much influenced by the coastline configurations and the shallow coastal seas and they are hardly representative of the mid-ocean tides. The best observational data of the latter comes from island stations that are little disturbed by local sea floor topography and all such records show that the undisturbed tide is little more than a meter. The available island measurements do not suffice for establishing the global pattern accurately, and the recent development of pressure tide gauges for measuring the tides in the open sea have made no impact yet on the global tide solutions. Our present knowledge of the open ocean tides comes from the more or less complete solutions of the Laplace tidal equations and a number of solutions for the $M_2$ tide (the principal semi-diurnal lunar tide) have been published recently. PEKERIS & ACCAD (1969) give solutions assuming a rigid Earth with, as boundary conditions, an impermeable coastline and allowing explicitly for dissipation in shallow seas. HENDERSHOTT (1972) allows for the effect of the tidal yielding of the solid Earth on the ocean tide and also attempts to evaluate the effect of the Earth's deformation under the variable ocean load (see also FARRELL 1972). Hendershott's boundary conditions are that the tide must correspond to the observed coastal values and dissipation is allowed for by allowing flow normal to the coastlines. The solutions of PEKERIS & ACCAD and of HENDERSHOTT agree in many areas but important discrepancies exist in, for example, the Pacific Ocean, pointing to the need for both improved theory and for more observational data. The $S_2$ ocean tide (the principal semi-diurnal solar tide) has been computed by BOGDANOV & MAGARIK (1967). No numerical solutions appear to exist for the other semi-diurnal tides or for the diurnal tides, although DIETRICH (1944) gives empirical cotidal charts for the $O_1$ (a nearly diurnal tide of solar origin) and $K_1$ (also nearly diurnal and of solar and lunar origin) tides as well as observed amplitudes along the coastlines and for some island sites.
3. Solid - Ocean Tide Interaction

The importance of the ocean tide interference with the solid tide has been demonstrated by the variable results obtained from surface measurements (for example, Kuo et al 1970; Pertsev 1969; Blum & Hatfield 1970; Berger & Lovberg 1970; Slichter 1972). The ocean loading of the continents appears to perturb all tide measurements, even those in the middle of the continents and although local tides are often most important (for example, Lambert 1970) even very distant tides will contribute to the observed combined tide (Kuo et al 1970; Pertsev 1969). In general, the ocean tides are not well enough known to be able to correct for this loading and improvements in the solid tide studies can only come about if there is also an improvement in our knowledge of the ocean tide (Hendershott & Munk 1970). We cannot separate fully, at present, the fluid and solid tides through lack of mathematical completeness of the ocean tide solutions and through lack of global observational ocean tide data in particular for the components other than the principal components. Progress in interpreting the solid tidal measurements in terms of phase lags, resonances or geological variations, can only be achieved by a concomitant progress in ocean tide solutions. It is possible to use the Earth tide measurements as constraints in these solutions in the sense that any departures from a theoretical response can be used as an integral of the ocean tide. The value of such constraints still has to be proved but in view of the numerical solutions extreme sensitivity to small changes in boundary conditions, it would seem probable that any additional constraints will be of value. The foregoing remarks are equally valid for tidal studies from terrestrial measurements as for tidal studies from satellite orbit analyses, the only difference being that the two provide different constraints and as such the two methods are entirely complementary (Lambeck et al 1973; 1974).

4. Satellite Methods for Tidal Studies

The tidal potential $\Delta U(r)$ at the satellite causes an additional force function that has to be taken into account when the satellite's equations of motion are integrated. This potential introduces perturbations in the motion of close Earth satellites and Kaula (1964; 1969) has given the necessary formalism. The frequencies of these tidal perturbations are governed by the frequencies of the satellite motion around the Earth and the perturbing body's motion in space and the principal perturbations will tend to group around the principal terms in the lunar and solar motion. Thus the semi-diurnal $M_2$ tide will give perturbations with periods near fourteen days and the solar $S_2$ tide will give perturbations with periods near six months. Also, as the satellite measures the integral effect of the tidal potential, the longer the period of the perturbation, the larger will be its amplitude. Thus the $S_2$ tide even though on the Earth's surface it has less than one half the amplitude of the $M_2$ tide it will cause perturbations in the satellite motion an order of magnitude larger than the $M_2$ perturbations. In the special cases where the satellite parameters and the Sun's or Moon's elements combine so as to give very long period orbital perturbations, tides that are very small on the Earth can give rise to very large orbital perturbations. The amplitudes of the tidal perturbations are proportional to the Love numbers. For most discussions of the tides, only the potential component of degree 2 is considered as the others are small. In particular, at the satellite height the potential decreases rapidly with increasing degree due to the term $(R/r)^{2\ell+1}$ in $\Delta U(r)$. Thus with the satellite methods it is $k_2$ that is observed. The difference between the observed phase of the perturbation and the phase of the perturbing potential assuming a perfectly elastic Earth, gives a measure of the phase lag. This lag has a value of at most one or two degrees
and is very much more difficult to observe with confidence than $k_2$. Some typical periods and amplitudes of the tidal orbital perturbations are given in Table 1.

Due to the ocean-continent distribution and the variable sea floor topography, the ocean tides, when expressed in terms of spherical harmonics will contain all harmonics of degree zero to infinity but the convergence appears to be rapid (see the solution of HENDERSHOTT 1972). This tide generates a potential which is readily expressed by a surface density layer representation and which gives an additional term to the force function acting on the satellite. We would expect that this ocean tide would generate perturbations of the same frequency as the solid tide due to that term in the ocean tide expansion that has the same degree and order as the solid tide. In addition, we could expect further perturbations resulting from the other harmonics in the ocean tide expansion. But most of these further perturbations are of short period - near the period of revolution of the satellite about the Earth - and as such do not build up into measurable perturbations (LAMBECK et al 1973). If the tidal potential is of degree 2 and order $m$, then the principal ocean tide perturbations, having the same frequencies as the solid tide perturbations, are caused by the ocean harmonics of degree and order $2,m$; $4,m$; $6,m$ etc, with a rapid decrease in importance. Table 2 gives some orders of magnitude. In general, the ocean perturbations are equal to about 10% of the solid tide. One observes therefore, a combined solid-ocean tide. Or, if the perturbation in an element $e$ due to the principal lunar tide is written as

$$\delta c_{st} = k_2 \phi \cos \gamma$$

the ocean tide perturbation with the same frequency $\gamma$ is

$$\delta c_{ot} = (c_{22} \phi_{22} + c_{42} \phi_{42} + c_{62} \phi_{62}) \cos \gamma$$

where the $c$ are coefficients in the ocean tide expansion (LAMBECK & CAZENAVE 1973; LAMBECK et al 1974). We observe

$$\delta c_{obs} = (k_2 \phi + c_{22} \phi_{22} + c_{42} \phi_{42} + ...) \cos \gamma$$

The factors $\phi$ and $\psi$ depend on the orbital parameters and one could imagine that a separation of the $k_2$ and $c$ is possible if different satellite orbits or different orbital elements are analysed. This is only partially true. The principal ocean term has exactly the same dependence on the orbital elements as the solid tide and a separation of $k_2$ and $c_{22}$ is not possible. A separation of these terms from $c_{42}$ (and eventually from $c_{62}$ if very precise tracking data becomes available) is possible and we can imagine an iterative procedure where we solve for

$$k_2 + \frac{3}{4} c_{22}, c_{42} \text{ and } c_{62}$$

and introduce the last two parameters as constraints in the solution of the Laplace tidal equations and compute the value $c_{22}$ and hence $k_2$. In exactly the same way, a separation of the phase lags resulting from the solid Earth and the oceans is not possible from the analysis of tidal perturbations alone and we can again envisage the above iterative approach.
5. Results

Love numbers have been estimated from orbital perturbations by KOZAI (1968), NEWTON (1968), ANDERLE (1971), DOUGLAS et al (1972), SMITH et al (1973) and LAMBECK et al (1974). Of these studies only the last considered the ocean tides. LAMBECK & CAZENAVE (1973) showed that the apparently aberrant results for Love numbers obtained by the various investigators resulted from their neglect of the ocean tide, and LAMBECK et al (1974) have applied the ocean corrections to the results of the earlier investigators to give a value for $k_2 = 0.306$ and a phase lag of 0.5 degrees (see Table 3). The latter leads to a mantle $Q$ of about 60, a value in reasonable agreement with seismic results (LAGUS & ANDERSON 1968). The studies of LAMBECK & CAZENAVE (1973) and LAMBECK et al (1973; 1974) lead to the following conclusions:

i) There is an important interaction between Earth and ocean tides as observed from orbit analyses. Neglect of the latter tides can introduce errors in $k_2$ of as much as 15% and of several degrees in phase;

ii) the ocean tide models, even the comparatively well known $M_2$ tide, are inadequate for making the precise ocean corrections, particularly for the important diurnal tides;

iii) the solid Earth tidal parameters computed from the satellite orbits are not yet very conclusive, even when corrected for the ocean tides. This is in part due to ii) but also due to residual non-tidal perturbations remaining in the satellite orbit parameters and due to inadequate tracking data. Better results can be expected in the future if precise laser tracking data, well distributed in space and time, can be collected from the already existing satellites and from the new small and dense satellite to be launched by the Centre National d'Etudes Spatiales in 1974 for gravimetric and tidal studies;

iv) a complete separation of fluid and solid tides is not possible from the analysis of satellite orbits alone, but the ocean tidal parameters that can be estimated can be used as constraints in the numerical tide solutions. This is also the case for the surface measurements of the bodily tide but in the former case the constraints, being the harmonics of degree 2 and 4 (and possibly 6 in the future) are of a global nature whereas the surface measurements provide constraints on the regional tides;

v) the ocean tide effect on the satellite orbit is frequency dependent (due to near resonances between the forcing function and the free periods of the oceans) and as such one must analyse the satellite orbits for specific tidal terms rather than solve for a single parameter that will present some average effect as has been done by SMITH et al (1973) and DOUGLAS et al (1972) as this leads to results that have no clear physical interpretation even though they may describe well the orbital perturbations of a particular satellite. This frequency dependence also makes the approach through latitude dependent Love numbers (KAULA 1969) impractical.

6. References


Table 1
Amplitudes and Periods of Perturbations in the Inclination of GEOS-1 and GEOS-2 due to some tidal Components

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Tide</th>
<th>Period (days)</th>
<th>( \Delta I ) (arcsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOS-1</td>
<td>( m_2 )</td>
<td>11.1</td>
<td>0.17</td>
</tr>
<tr>
<td>GEOS-1</td>
<td>( s_2 )</td>
<td>55.7</td>
<td>0.40</td>
</tr>
<tr>
<td>GEOS-1</td>
<td>( k_3 )</td>
<td>160.7</td>
<td>0.95</td>
</tr>
<tr>
<td>GEOS-2</td>
<td>( m_2 )</td>
<td>15.3</td>
<td>0.30</td>
</tr>
<tr>
<td>GEOS-2</td>
<td>( s_2 )</td>
<td>43.2</td>
<td>4.03</td>
</tr>
</tbody>
</table>

Table 2
Amplitudes of the Orbital Perturbations in I and \( \Omega \) due to the Second and Fourth Harmonics in the Ocean \( m_2 \) Tide and Compared with the Corresponding Earth Tide

<table>
<thead>
<tr>
<th>Inclination I</th>
<th>Ascending Node ( \Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth Tide ( m_2 )</td>
<td>Ocean Tide ( m_2 ) Second Harmonic</td>
</tr>
<tr>
<td>70t0901 (PEO)</td>
<td>0°06</td>
</tr>
<tr>
<td>6503201 (BE-C)</td>
<td>0°16</td>
</tr>
<tr>
<td>6508901 (GEOS-1)</td>
<td>0°19</td>
</tr>
<tr>
<td>6406401 (BE-B)</td>
<td>0°31</td>
</tr>
<tr>
<td>6402601 (TRANSIT)</td>
<td>0°33</td>
</tr>
<tr>
<td>6800201 (GEOS-2)</td>
<td>0°32</td>
</tr>
</tbody>
</table>

Table 3
Summary of Results Obtained by Other Authors and Corrected for Ocean Tidal Parameters

<table>
<thead>
<tr>
<th>Author</th>
<th>Satellite</th>
<th>Tidal Element</th>
<th>( k_2 ) Analysed</th>
<th>( k_2 ) Corrected</th>
<th>( \delta_2 ) Observed</th>
<th>( \delta_2 ) Corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kozai</td>
<td>5900101</td>
<td>( K^m + K^s )</td>
<td>0.22</td>
<td>0.24</td>
<td>-5°5</td>
<td>-1°5</td>
</tr>
<tr>
<td>Kozai</td>
<td>6000902</td>
<td>( K^m + K^s )</td>
<td>0.31</td>
<td>0.34</td>
<td>1°3</td>
<td>3°7</td>
</tr>
<tr>
<td>Kozai</td>
<td>6206001</td>
<td>( K^m + K^s )</td>
<td>0.32</td>
<td>0.34</td>
<td>0°7</td>
<td>6°7</td>
</tr>
<tr>
<td>Newton</td>
<td>Mean of four Polar Satellites</td>
<td>( m_2 )</td>
<td>( \Omega )</td>
<td>0.29</td>
<td>0.32</td>
<td>1°7</td>
</tr>
<tr>
<td>Newton</td>
<td>( S_2 )</td>
<td>( \Omega )</td>
<td>0.34</td>
<td>0.36</td>
<td>1°6</td>
<td>-3°4</td>
</tr>
<tr>
<td>Newton</td>
<td>( S_2 )</td>
<td>( \Omega )</td>
<td>0.33</td>
<td>0.36</td>
<td>1°2</td>
<td>-3°8</td>
</tr>
<tr>
<td>Douglas et al.</td>
<td>6508901</td>
<td>( k_2 + S_2 )</td>
<td>0.22</td>
<td>0.25</td>
<td></td>
<td></td>
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<tr>
<td>et al.</td>
<td>6800201</td>
<td>( k_2 + S_2^{+P} )</td>
<td>0.31</td>
<td>0.33</td>
<td></td>
<td></td>
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<td>6502801</td>
<td>( k_2 + S_2^{+P} )</td>
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<td>0.28</td>
<td>3°2</td>
<td>5°</td>
</tr>
<tr>
<td>Lamebeck et al.</td>
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<td>( m_2 )</td>
<td>( \Omega )</td>
<td>0.29</td>
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<td>9°</td>
</tr>
</tbody>
</table>

Arithmetic Mean | 0.309 | 0°5 |


