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Effects of Tidal Dissipation in the Oceans on the Moon's Orbit and the Earth's Rotation

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The dissipation of tidal friction in the oceans and atmosphere has been estimated by calculating the secular perturbations of the moon's orbit by the tides using the available tide models for the principal semidiurnal and diurnal tides. Only the second-degree wavelength components in the ocean tide cause significant secular changes in the lunar orbit. These components of the M_2 tide agree within 15% among the different ocean tide models. Other frequencies (O_1, N_2, S_2, P_1, and K_1) have effects on the lunar motion of about 25% of the M_2 tide and on the earth's rotation of about 45% of M_2. The computed lunar acceleration of −35 arc sec/(100 yr)^2 agrees to better than 15% with nearly all recent astronomical determinations. There has been no significant change in the tidal effect during the last 2000 or 3000 years. The difference between the two estimates of the lunar acceleration is of the same order as their uncertainties. Hence only upper limits can be set on the $1/Q$ of the solid earth and moon of about 1/50 and on dissipation in the core of about 3×10^{18} ergs s^{-1}. The total tidal acceleration of the earth is estimated to result in a 3.7-ms/100 yr increase in the length of day. The most recent discussion of the ancient eclipse records indicates an observed increase in the length of day of only 2.5 ms/100 yr. If the nontidal acceleration is attributed to the postglacial response of the earth, the decay time is about 3000 years, which is equivalent to a mean viscosity for the mantle of about 10^{22} P.

INTRODUCTION

Tidal dissipation is the recognized explanation for the observed secular acceleration of the moon and the deceleration of the earth's rotation, but the energy sink has not been unambiguously identified [*Munk and MacDonald,* 1960; *Jeffreys,* 1962]. It is generally assumed that the world's oceans must play a dominant role in the dissipation of tidal energy, but the most recent calculations of dissipation in the oceans by *Miller* [1966], *Pekeris and Accad* [1969], and *Hendershott* [1972] are not entirely conclusive. Part of the total dissipation may also occur in the solid parts of the earth, and an attempt at separating these two possible energy sinks is of interest insofar as it would give an estimate of the solid earth's global anelastic properties, namely, the specific dissipation function ($1/Q$), at low frequencies, and would provide further insight into the evolution of the lunar orbit and the earth's rotation.

To calculate the tidal dissipation in the world's oceans, three approaches are possible [*Munk and MacDonald,* 1960; *Kaula,* 1968]. The first is to calculate the mean rate of work per unit area done by the sun and moon on the ocean surface. The second is to calculate the energy flux per unit time and per unit area across the entrances to shallow seas. The third is to calculate the rate of work per unit surface done by currents on the sea floor. *Miller* [1966] used essentially the second approach, which, because it depends only on the first power of the tidal velocity, is usually considered to be the most reliable. The third method depends on the third power of the velocity. The first method has been used by several investigators, including *Groves and Munk* [1958] and most recently *Pariyskiy et al.* [1972] and *Kuznetsov* [1972]. This method has the advantage of being independent of the nature of the ocean floor dissipation mechanism and does not require a knowledge of the ocean currents. Neither does it assume that friction occurs in shallow seas only. But it does require a knowledge of the tide over the entire ocean surface. Nevertheless, we use this approach here because, as far as the effect of tidal dissipation on the moon is concerned, only certain wavelengths in the ocean tide are important and these wavelengths seem to be relatively well known for the major tides. Thus our approach differs from the standard method in that the rates of work done by the moon and sun on these long wavelengths, rather than on the total ocean tide, are integrated over the ocean surface. *Kaula* [1969] also used the first approach implicitly by developing the tidal effective Love numbers and the lag angles as functions of latitude, but his approach does not lend itself to a ready interpretation in terms of the usual description of the ocean tides. Also, Kaula does not allow for the frequency dependence of these global tidal parameters. The development given here follows that of *Lambeck et al.* [1974] for the similar problem of the orbital evolution of close earth artificial satellites acted upon by the potential of the earth's tides.

TIDAL POTENTIALS

Solid earth tides. The earth, acted upon by an external force, will deform, and this deformation will result in a further perturbing potential ΔU outside the earth that will contribute to the total force function acting upon the satellite. This additional potential can be expressed in terms of the equatorial Keplerian elements $a^*e^*i^*M^*\omega^*\Omega^*$ of the tide-raising object (sun or moon) and of the satellite position $aeiM\omega\Omega$, at which ΔU is evaluated, in the following way [*Kaula,* 1964; *Lambeck et al.,* 1974]:

$$\Delta U = \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{p=0}^{l} \sum_{q=-\infty}^{\infty} \sum_{j=0}^{l} \sum_{g=-\infty}^{\infty} k_l \left(\frac{R}{a^*}\right)^l \left(\frac{R}{a}\right)^{l+1} \frac{Gm^*}{a^*} \cdot \frac{(l-m)!}{(l+m)!} (2 - \delta_{om}) F_{lmp}(i^*) G_{lpq}(e^*) F_{lmj}(i) G_{ljg}(e) \cdot \cos (v_{lmpq}^* - v_{lmjg} + \epsilon_{lmpq}) \quad (1)$$

with

$$v_{lmpq}^* = (l-2p)\omega^* + (l-2p+q)M^* + m\Omega^*$$

and

$$v_{lmjg} = (l-2j)\omega + (l-2j+g)M + m\Omega$$

where m^* is the mass of the tide-raising object, R is the mean radius of the earth, and k_l is the Love number of degree l. The phase angle ϵ_{lmpq} expresses the anelastic response of the earth and is related to the time delay Δt in the response by [*Lambeck et al.*, 1974]

$$\epsilon_{lmpq} = \upsilon_{lmpq}{}^* - \dot{\upsilon}_{lmpq}{}^* \Delta t \approx -(l - 2p + q)\dot{M}^* \Delta t + m\dot{\theta} \Delta t$$

The orbital elements $aeiM\omega\Omega$ will vary in time owing inter alia to the tidal potential (1). We are concerned here with the effect of this potential on the lunar orbit itself and in particular with the secular evolution of the orbit. In the absence of tidal friction the conservative elements of the earth-moon motion are [*Goldreich*, 1966] (1) the scalar angular momentum of the lunar orbital motion, (2) the scalar angular momentum of the earth's spin, (3) the component normal to the ecliptic of the total angular momentum in the earth-moon system, and (4) the sum of the potential energies. In the presence of tidal dissipation these elements will vary on the long time scale, and these changes are related to the secular changes in the lunar orbit's inclination, eccentricity, and semimajor axis. The time variations in these elements can be found upon substituting the potential ΔU into the Lagrangian planetary equations. Next, to estimate the effect on the lunar orbit, we can equate $a^*e^*i^*M^*\omega^*\Omega^*$ with $aeiM\omega\Omega$. Finally, as we are interested in the secular changes, we require only terms of zero frequency: $\dot{\upsilon}_{lmpq}{}^* - \dot{\upsilon}_{lmjg} = 0$, or $p = j$ and $q = g$ [*Kaula*, 1964]. The secular changes in the lunar inclination i, eccentricity e, and semimajor axis a are then given by

$$\dot{a}_{lmpq} = 2K_{lm}[F_{lmp}(i)]^2[G_{lpq}(e)]^2(l - 2p + q) \sin \epsilon_{lmpq}$$

$$\dot{e}_{lmpq} = K_{lm} \frac{(1 - e^2)^{1/2}}{ae} [F_{lmp}(i)]^2[G_{lpq}(e)]^2[(1 - e^2)^{1/2} \cdot (l - 2p + q) - (l - 2p)] \sin \epsilon_{lmpq}$$

$$\left(\frac{di}{dt}\right)_{lmpq} = K_{lm} \frac{[(l - 2p) \cos i - m]}{a(1 - e^2)^{1/2} \sin i} \cdot [F_{lmp}(i)]^2[G_{lpq}(e)]^2 \sin \epsilon_{lmpq}$$

with

$$K_{lm} = \frac{Gm^*k_l}{[G(M + m^*)a]^{1/2}} \left(\frac{R}{a}\right)^{2l+1} \frac{(l - m)!}{(l + m)!} (2 - \delta_{om})$$

We have ignored any contributions that could arise from the dissipation of energy due to tides raised on the moon by the earth. Substituting into (1) the values of the planet parameters for those of the corresponding lunar parameters gives the potential of the moon's tidal deformation per unit mass of the planet. The effect on the lunar orbit relative to the planet is then found by multiplying this potential by the factor M/m^*, where M is the mass of the planet and m^* is the mass of the satellite [*Kaula*, 1964].

The principal perturbations in the lunar orbit due to the potential are given in Table 1. To identify them with the usual nomenclature of the ocean tides, we also give the equivalences between the orbital indices $lmpq$ and the Darwinian nomenclature [*Lambeck et al.*, 1974]. The major contribution to $\dot{a}_{lmpq}$ comes from the M_2 tide, as has been understood since the work of G. H. Darwin. The K_1 tide does not perturb the semimajor axis, since $l - 2p + q = 0$. The diurnal O_1 tide represents about 20% of the effect of M_2 and N_2, about 5%. The sum of all other diurnal and semidiurnal tides contributes less than 1% of the M_2 contribution. The fortnightly tide corresponds to the sum of the terms with $lmpq$ = 2000 and $lmpq$ = 2020, for which the phase angles are $-2\dot{M}^*\Delta t$ and $+2\dot{M}^*\Delta t$, respectively. Its contribution to $\dot{a}$ is about 1%, and that of the monthly tide even less. For the eccentricity the principal secular change comes from the N_2 tide with significant contributions from M_2, L_2, O_1, and M_m. For di/dt the contributions of the M_2, K_1, and O_1 tides are of similar magnitude, but the last two are of opposite sign and will largely cancel.

Ocean tides. The global ocean tides are known approximately only for the principal tides, such as M_2, S_2, K_1, and O_1. These tides are expressed at any point in the world's ocean by an amplitude ξ_0 and phase β as

$$\xi_\mu(\phi, \lambda) = \xi_0(\phi, \lambda) \cos [2\pi fT - \beta(\phi, \lambda)]$$

where f is the frequency of the tidal component μ being analyzed. On the continents, $\xi = 0$. For the available ocean

TABLE 1. Amplitudes of the Secular Variations in *aei* of the Lunar Orbit Due to the Principal Frequencies in the Solid Earth Tide

lmpq	Tidal Component μ	Multiplying Factor*	*da/dt*	*de/dt*	*di/dt*
2200	M_2	$\frac{3}{16}(1 + \cos i)^4 \frac{X}{a}$	$a(1 - 5e^2)$	$-\frac{1}{4}e$	$\frac{1}{2}\left(1 - \frac{9e^2}{2}\right)\frac{\cos i - 1}{\sin i}$
2201	N_2	$\frac{147}{32}(1 + \cos i)^4 \frac{X}{a}$	$\frac{3}{4}ae^2$	$\frac{1}{8}e$	$\frac{1}{4}e^2\left(\frac{\cos i - 1}{\sin i}\right)$
220(−1)	L_2	$\frac{3}{32}(1 + \cos i)^4 \frac{X}{a}$	$\frac{1}{4}ae^2$	$-\frac{1}{8}e$	$\frac{1}{4}e^2\left(\frac{\cos i - 1}{\sin i}\right)$
2110	K_1	$\frac{3}{4}(\sin^2 i \cos^2 i) \frac{X}{a}$	0	0	$-\left(1 + \frac{7e^2}{2}\right)\frac{1}{\sin i}$
2100	O_1	$\frac{3}{4} \sin^2 i (1 + \cos i)^2 \frac{X}{a}$	$(1 - 5e^2)a$	$-\frac{1}{4}e$	$\frac{1}{4}\left(1 - \frac{9e^2}{2}\right)\frac{(2 \cos i - 1)}{\sin i}$
2000	M_f	$(\frac{9}{16} \sin^4 i) \frac{X}{a}$	$(1 - 5e^2)a$	$\frac{1}{4}e$	$\frac{1}{2}\left(1 - \frac{e^2}{2}\right)\frac{\cos i}{\sin i}$
2020			$-(1 - 5e^2)a$	$-\frac{1}{4}e$	$-\frac{1}{2}\left(1 - \frac{e^2}{2}\right)\frac{\cos i}{\sin i}$
2011	M_m	$\frac{9}{16}(1 - 3 \sin^2 i + \frac{9}{4} \sin^4 i) \frac{X}{a}$	e^2a	e	0
201(−1)			$-e^2a$	$-e$	0

* $X = \{k_2Gm/[aG(M + m)]^{1/2}\}(R/a)^6 \sin \epsilon_{lmpq}$.

models the ξ_0 and β have been evaluated at grid intervals of 10° in latitude and longitude and the $\xi_0 \cos \beta$ and $\xi_0 \sin \beta$ are expanded in series of spherical harmonics [*Lambeck and Cazenave,* 1973]. The tide can then be expressed by

$$\xi_\mu = \sum_{s=0}^{\infty} \sum_{t=0}^{s} \sum_{+}^{-} P_{st}(\sin \phi) \cdot (C_{st}^{\pm})_\mu \sin [2\pi f T \pm t\lambda + (\epsilon_{st}^{+})_\mu]$$

where the $(C_{st}^{\pm})$ and $(\epsilon_{st}^{\pm})_\mu$ are functions of the spherical harmonic expansions of the μ tide component. The potential expressed in Keplerian elements outside the earth due to this layer of density ρ_ω is

$$\Delta U_\mu' = \frac{4\pi G R^2}{a} \sum_{s=0}^{\infty} \sum_{t=0}^{s} \sum_{u=0}^{s} \sum_{v=-\infty}^{\infty} \sum_{+}^{-} \frac{1 + k_s'}{2s + 1} \cdot \left(\frac{R}{a}\right)^s \rho_\omega (C_{st}^{\pm})_\mu F_{stu}(i) G_{suv}(e) \begin{bmatrix} \sin \\ -\cos \end{bmatrix}_{s-t \text{ odd}}^{s-t \text{ even}} \gamma_{stuv}^{\pm} \qquad (2)$$

with

$$\gamma_{stuv}^{\pm} = (s - 2u)\omega + (s - 2u + v)M + t(\Omega - \theta) \pm 2\pi f T + (\epsilon_{st}^{\pm})_\mu$$

The k_s' are the load deformation coefficients of degree s and allow for the deformation of the earth under the variable ocean load. For the secular effects of $\Delta U_\mu'$ on the lunar orbit we require that $\dot{\gamma}_{stuv}^{\pm} = 0$. The correspondence between the equatorial elements $M\omega\Omega$ and the ecliptic elements in which $2\pi fT$ is usually expressed is discussed by *Lambeck et al.* [1974]. For the M_2 tide, for example, $\dot{\gamma}^{\pm}$ vanishes for $\dot{\gamma}^{+}$ only, and then when $t = m$, $s - 2u = t$, $s - 2v + v = t$, and $v = 0$; or when $s = 2$, $u = 0$; when $s = 4$, $u = 1$; when $s = 6$, $u = 2$; etc. Because of the $(R/a)^s$ term in $\Delta U_\mu'$ the principal contributions will occur for $s = 2$. For close earth satellites, perturbations due to the coefficients of degree 4 can be important, but for the lunar orbit these perturbations will be smaller than those due to the coefficients of degree 2 by a factor of $(1/60)^2$. The secular perturbations in the lunar orbit are given by

$$\dot{a} = 2 K_{stuv}'(s - 2u + v) \begin{bmatrix} \cos \\ \sin \end{bmatrix}_{s-t \text{ odd}}^{s-t \text{ even}} (\epsilon_{st}^{+})_\mu$$

$$\dot{e} = K_{stuv}' \frac{(1 - e^2)^{1/2}}{ae} [(1 - e^2)^{1/2}(s - 2u + v) - (s - 2u)] \begin{bmatrix} \cos \\ \sin \end{bmatrix}_{s-t \text{ odd}}^{s-t \text{ even}} (\epsilon_{st}^{+})_\mu$$

$$\frac{di}{dt} = K_{stuv}' \frac{[(s - 2u) \cos i - t]}{a \sin i \, (1 - e^2)^{1/2}} \begin{bmatrix} \cos \\ \sin \end{bmatrix}_{s-t \text{ odd}}^{s-t \text{ even}} (\epsilon_{st}^{+})_\mu \qquad (3)$$

with

$$K_{stuv}' = \frac{3GM}{[G(M + m^*)a]^{1/2}} \frac{1 + k_s'}{2s + 1} \frac{\rho_\omega}{\bar{\rho}} \left(\frac{R}{a}\right)^s \cdot (C_{st}^{+})_\mu F_{stu}(i) G_{suv}(e)$$

where $\bar{\rho}$ is the mean density of the earth.

Table 2 summarizes the principal results for the ocean tide perturbations, and these results can be compared directly with the solid tide perturbations in Table 1. The relative importance of the two types of perturbations will depend largely on the phase angles occurring in the two potentials.

Table 3 summarizes the relevant coefficients $(C_{st}^{\pm})_\mu$ and $(\epsilon_{st}^{\pm})_\mu$ for some of the available ocean tide models. For M_2, results of three models are available. *Hendershott* [1972] gives directly coefficients of the spherical harmonic expansion that can be related to the $(C_{st}^{\pm})_\mu$ and $(\epsilon_{st}^{\pm})_\mu$. The other models used are those of *Bogdanov and Magarik* [1967] and *Pekeris and Accad* [1969]. Major regional discrepancies occur between these solutions (see the discussion by *Hendershott* [1973]), but the three individual estimates of $(C_{22}^{+} \cos \epsilon_{22}^{+})_\mu$ differ from their mean by only 10%. For K_1 and O_1 the interpolated solutions of *Dietrich* [1944], as discussed by *Lambeck et al.* [1974], are used. Since these calculations a numerical solution for the K_1 tide has been published by *Zahel* [1973]. For the relatively important N_2 tide, for which we have neither numerical nor empirical models, we assume that $(C_{22}^{+})_{M_2}$ and $(C_{22}^{+})_{N_2}$ are proportional to the corresponding tide-raising potentials, or

$$(C_{22}^{+})_{N_2} \approx (C_{22}^{+})_{M_2} \cdot \frac{\Delta U_{2201}}{\Delta U_{2200}} \approx \frac{7}{2} e (C_{22}^{+})_{M_2}$$

and $(\epsilon_{22}^{+})_{N_2} \approx (\epsilon_{22}^{+})_{M_2}$ in view of the fact that the frequencies of M_2 and N_2 are similar and that the solutions by *Bogdanov and Magarik* [1967] for the M_2 and S_2 tides show a similar proportionality. For L_2 with $lmpq = 220(-1)$ we have $\Delta U_{220(-1)} = (-e/2) \Delta U_{2220}$, and we assume that

$$(C_{22}^{+})_{L_2} \approx (-e/2)(C_{22}^{+})_{M_2}$$

and

$$(\epsilon_{22}^{+})_{L_2} \approx (\epsilon_{22}^{+})_{M_2}$$

For the fortnightly tide M_f and the monthly tide M_m no tidal information is available on a global scale.

EVOLUTION OF THE LUNAR ORBIT

Secular changes in the semimajor axis. We introduce the subscripts M and S to refer to lunar and solar parameters, respectively. Table 4 summarizes the computed $\dot{a}_M$ for the principal ocean models. The three estimates obtained for the M_2 tide differ by a maximum of 15%. In the subsequent calcula-

TABLE 2. Amplitude of the Secular Variations in *aei* of the Lunar Orbit Due to the Principal Frequencies in the Ocean Tide

lmpq	Tide μ	Multiplying Factor*	*da/dt*	*de/dt*	*di/dt*
2200	M_2	$\frac{Z}{a}(1 + \cos i)^2 \cos (\epsilon_{22}^{+})_\mu$	$\frac{9}{5}a\left(1 - \frac{5e^2}{2}\right)$	$-\frac{9}{20}e$	$\frac{9}{10}(1 + 2e^2)\left(\frac{\cos i - 1}{\sin i}\right)$
2201	N_2	$\frac{Z}{a}(1 + \cos i)^2 \cos (\epsilon_{22}^{+})_\mu$	$\frac{189}{20}ea$	$\frac{63}{40}(1 - 2e^2)$	$\frac{63}{20}e\left(\frac{\cos i - 1}{\sin i}\right)$
220(−1)	L_2	$\frac{Z}{a}(1 + \cos i)^2 \cos (\epsilon_{22}^{+})_\mu$	$-\frac{9}{20}ea$	$\frac{9}{40}$	$-\frac{9}{20}e\left(\frac{\cos i - 1}{\sin i}\right)$
2110	K_1	$\frac{Z}{a}\cos i \sin i \sin (\epsilon_{21}^{+})_\mu$	0	0	$\frac{9}{10}\left(\frac{1 + 2e^2}{\sin i}\right)$
2100	O_1	$\frac{Z}{a}\sin i (1 + \cos i) \sin (\epsilon_{21}^{+})\mu$	$\frac{9}{5}\left(1 - \frac{5e^2}{2}\right)a$	$-\frac{9}{20}e$	$\frac{9}{20}(1 - 2e^2)\left(\frac{2 \cos i - 1}{\sin i}\right)$

* $Z = \{(1 + k_2')/[G(M + m)a]^{1/2}\}(GM/R)(\rho\omega/\rho)(R)^2/(a)(C_{lm}^{+})$.

TABLE 3. Amplitudes and Phases of the Principal Components in the Fluid Tides Leading to Secular Variations in *aei* of the Lunar Orbit

Tide μ	Solution	$(C_{2m}^{+})_\mu$, cm	$(\epsilon_{2m}^{+})_\mu$, deg
M_2	*Pekeris and Accad* [1969]	4.4	340
M_2	*Hendershott* [1972]	5.1	316
M_2	*Bogdanov and Magarik* [1967]	4.3	318
S_2	*Bogdanov and Magarik* [1967]	1.6	332
K_1	*Dietrich* [1944]	3.3	318
O_1	*Dietrich* [1944]	1.8	142
P_1	Estimated	0.7	140
N_2	Estimated	0.9	320
L_2	Estimated	−0.1	330
S_2	Atmospheric tide*	0.34	158

* Converted to an equivalent water layer.

tions the mean of the three models is used. The secular change in the mean motion n_M of the moon is related to $\dot{a}_M$ by

$$\dot{n}_M = -3/2(n_M/a_M)\dot{a}_M \qquad (4)$$

and is also given in Table 4. We have ignored the tide raised on the moon by the earth's attraction, and the possibility remains that the tidal dissipation in the moon may be important. The M_2 tide will not lead to dissipation in the moon, since it raises a permanent tide on the moon. Dissipation in the moon will occur due to the O_1 and N_2 tides, but the actual amount will be small. From equation (48) of *Kaula* [1964] we obtain

$$\frac{(\dot{a})_{\text{moon effect}}}{(\dot{a})_{\text{earth effect}}} = \left(\frac{M}{m_M}\right)^2\left(\frac{R_M}{R}\right)^5 \cdot \frac{(k_2)_M \sin \epsilon_M}{k_2 \sin \epsilon}$$

with $(k_2)_M = 0.020$ [*Kaula,* 1968]. The phase angle ϵ_M is unknown. Seismic data from the Apollo seismic network indicate a Q for shear waves for the upper 500–600 km of the moon higher than several thousand and below this depth a Q of about 300 with a central region Q of about 100 [*Toksöz et al.,* 1974]. We take $Q = 150$ and $\epsilon_M = 1°$, as compared with the observed ϵ of the earth about 7.5° (see discussion below). The moon effect on $\dot{a}_M$ for the O_1 and N_2 tides will then be of the order of 0.05×10^{-7} cm s^{-1} or smaller and unimportant.

Secular changes in eccentricity and inclination. Table 5 summarizes the secular changes in the eccentricity of the lunar orbit due to dissipation in the oceans. The total estimated rate of change in eccentricity cannot be very certain in view of the importance of the N_2 tide, for which no direct estimates of C_{22}^{+} and ϵ_{22}^{+} are available. An important contribution to $\dot{e}_M$ can come from the N_2 tide raised on the moon, but the consequences of this on the lunar orbit evolution cannot be evaluated without a reliable value for the Q of the moon at the tidal frequency [*Goldreich,* 1966]. The secular changes in the inclination of the lunar orbit on the equatorial plane due to the ocean tides are also given in Table 5. The tides on the moon do not appear to be important, since the effects of K_1 and O_1 largely cancel. The total computed present value of di/dt is small, less than $2°/10^9$ years.

Astronomical observations for the lunar acceleration. The astronomically observed quantity is the non-Newtonian secular acceleration of the moon's mean longitude and is due in part to the earth's variable rate of rotation $\dot{\omega}$ and in part to the tidal acceleration of the moon, $\dot{n}_M$. We ignore any possible contribution from a secular change in the gravitational constant. The $\dot{n}_M$ and $\dot{\omega}$ are separated by combining the observed lunar acceleration with observed solar or planetary accelerations or by observing the earth's rotation independently of the lunar motion. Most previous discussions of the past evolution of the lunar orbit use the accelerations of Spencer-Jones or of Fotheringham [*Munk and MacDonald,* 1960], but a number of recent and independent analyses indicate that the accelerations deduced from these results are in error by a factor of almost 2. *Newton* [1970] analyzed ancient eclipse observations and found $\dot{n}_M = 41.6 \pm 4$ arc sec/(100 yr)^2 near 200 B.C. and −42.4 ± 6 arc sec/(100 yr)^2 near 1000 A.D. The more recent interpretation by *Muller and Stephenson* [1975] of the ancient eclipse observations gives −37.5 ± 5 arc sec/(100 yr)^2. From

TABLE 4. Summary of Computed Tidal Variations in the Lunar Orbit and in the Earth's Secular Rotation

Tide	Solution	da/dt, 10^{-7} cm s^{-1}	dn/dt, 10^{-23} rad s^{-2}	$\dot{\omega}$, 10^{-22} rad s^{-2}: $\dot{\omega}_{T\mid a}$	$\dot{\omega}_{T\mid e}$	$\dot{\omega}_{T\mid i}$	$\dot{\omega}_T$
Lunar							
M_2	*Hendershott* [1972]	1.41	−1.47				
M_2	*Pekeris and Accad* [1969]	1.59	−1.65				
M_2	*Bogdanov and Magarik* [1967]	1.33	−1.28				
M_2	Mean	1.41	−1.47	−6.04	−0.01	−0.56	−6.61
O_1	*Dietrich* [1944]	0.10	−0.11	−0.46		0.17	−0.29
N_2	Estimated	0.09	−0.10	−0.41	0.10	−0.02	−0.34
L_2	Estimated				−0.02		−0.02
K_1	Estimated					−0.40	−0.40
Solar							
S_2	*Bogdanov and Magarik* [1967]						−1.34
P_1	Estimated						−0.05
R_2	Estimated						−0.07
Atmospheric							
S_2	Kertz et al.*						0.26
Total		1.60	−1.68	−6.91	−0.13	−0.81	−8.86

$\dot{\omega}_{T\mid a}$ is the contribution from the secular change in a, $\dot{\omega}_{T\mid e}$ that from e, and $\dot{\omega}_{T\mid i}$ that from i.

* Quoted in *Chapman and Lindzen* [1970].

occultations of stars by the moon since 1955, *Van Flandern* [1970] determined $\dot{n}_M = -52 \pm 4$ arc sec/(100 yr)^2; from meridian circle observations of the sun, moon, and planets since 1913, *Oesterwinter and Cohen* [1972] found $\dot{n}_M = -38 \pm 8$ arc sec/(100 yr)^2, and from lunar occultation data since 1663, *Morrison* [1973] estimated $\dot{n}_M = -42 \pm 6$ arc sec/(100 yr)^2. The results from the eclipse, occultation, and meridian observations are in good agreement. There is no evidence that any significant change in the tidal effects has occurred during the last 2000 years. The ocean tidal estimate of $\dot{n}_M$ is, from Table 4 (see also the comparison Table 6), -35 ± 4 arc sec/(100 yr)^2, in agreement with the astronomical estimates. In contradiction to *Newton*'s [1970, p. 387] argument, a value of $\dot{n}_M$ of about -40 arc sec/(100 yr)^2 is in complete harmony with the energy dissipation within the earth-moon system. In particular, as the tide models used depend on recent tide observations, the agreement between the tidal and the astronomical estimates is further evidence that there has been no significant change in the rate of dissipation of tidal energy between about 500 B.C. and the present. We can conclude with confidence that if not all, at least a very major part of the secular change in the moon's mean longitude, is caused by dissipation of tidal energy in the oceans, and we do not have to invoke significant energy sinks in the earth's mantle or core.

Earth's Secular Acceleration

For a circular orbit the angular momentum of the orbital motion of the moon and earth about their center of mass is

$$H_M = [Mm_M/(M + m_M)]a_M^2 n_M(1 - e_M^2)^{1/2} \qquad (4')$$

The angular momentum of the orbital motion of the earth and sun about their center of mass is

$$H_S = [Mm_S/(M + m_S)]a_S^2 n_S(1 - e_S^2)^{1/2} \qquad (4'')$$

The component of H parallel to the rotation axis is $H \cos i$. The angular momentum of the earth's rotation is $C\omega$, where C is the moment of inertia about the spin axis. When it is assumed that the angular momenta associated with the spin of the moon and sun are negligibly small, that the inertia tensors of the three bodies do not vary with time, and that G is constant, the change in the earth's acceleration $\dot{\omega}_T$ due to the tidal potential is

$$\dot{\omega}_T = \dot{\omega}_{T_M} + \dot{\omega}_{T_S} = -\frac{1}{C}(L_M + L_S)$$

$$= -\frac{d}{dt}(H_M \cos i_M + H_S \cos i_S)$$

where the $-L_M$ and $-L_S$ represent the lunar and solar tidal torques acting on the earth. With (4′),

$$\dot{\omega}_{T_M} = -\frac{1}{C}\frac{Mm_M}{M + m_M}\frac{d}{dt}[a_M^2 n_M(1 - e_M^2)^{1/2} \cos i_M]$$

$$\approx \frac{1}{C}\frac{Mm_M}{M + m_M} n_M a_M^2\left(\frac{1}{3}\cos i_M \frac{\dot{n}_M}{n_M} + e_M \cos i_M \dot{e}_M + \sin i_M \frac{di_M}{dt}\right) \qquad (5a)$$

The astronomically observed quantity is the $\dot{n}_M$. Thus the first term of (5*a*),

$$\dot{\omega}_{T_M|a_M} = \frac{1}{3C}\frac{Mm_M}{M + m_M} a_M^2 \cos i_M \dot{n}_M$$

can be evaluated directly from the astronomical observations or from those ocean tides raised by the moon that transfer angular momentum through a change in a_M or n_M. Table 4 summarizes the computed $\dot{\omega}_{T_M}$ for the principal tidal

TABLE 5. Computed Secular Variations in the Lunar Eccentricity and Inclination on the Equator Due to the Principal Ocean Tides

Ocean Tide	de/dt, s^{-1}	di/dt, rad s^{-1}
M_2	-4.90×10^{-20}	-3.85×10^{-19}
N_2	6.00×10^{-19}	-1.39×10^{-20}
L_2	-1.08×10^{-20}	-2.50×10^{-21}
O_1	-3.19×10^{-21}	1.19×10^{-19}
K_1	0	-2.74×10^{-19}
Total	5.36×10^{-19}	-5.57×10^{-19}

TABLE 6. Summary of the Observed Lunar Acceleration $\dot{n}_M$, $\dot{a}_M$, the Earth's Acceleration $\dot{\omega}$, and the Energy Dissipated $\dot{E}$

Solution	$\dot{n}$, 10^{23} rad s^{-2}	$\dot{a}$, 10^{-7} cm s^{-1}	$\dot{\omega}$, 10^{-22} rad s^{-2}: Tides*	Tides†	Observed	Nontidal	$\dot{E}$, 10^{19} ergs s^{-1}
1. Spencer-Jones¶	−1.09	1.04					
2. *Newton* [1970]‡	−2.04 ± 0.24	1.98	−8.36 ± 1.12	−10.77 ± 1.24	−5.80 ± 0.7	4.97 ± 1.42	6.03 ± 0.55
3. *Newton* [1970]§					−6.52 ± 0.7	4.25 ± 1.42	
4. *Muller and Stephenson* [1975]	−1.81 ± 0.24	1.76	−7.39 ± 1.07	−9.80 ± 1.12	−6.70 ± 0.7	3.10 ± 1.32	5.59 ± 0.50
Best mean solution of Muller and Stephenson	−1.96 ± 0.12	1.91	−8.03 ± 0.54	−10.44 ± 0.64	−7.05 ± 0.35	3.39 ± 0.73	5.84 ± 0.26
5. *Oesterwinter and Cohen* [1972]	−1.85 ± 0.38	1.77	−7.49 ± 1.70	−9.90 ± 1.73			5.54 ± 0.75
6. *Morrison* [1975]	−2.02 ± 0.29	1.96	−8.26 ± 1.30	−10.67 ± 1.34	−4.01 ± 1.0	6.66 ± 1.67	6.08 ± 0.59
Mean of solutions 4–6	−1.9 ± 0.2	1.8	−7.68 ± 1.0	−10.09 ± 1.0			5.7 ± 0.50
Tidal estimation (Table 4)	−1.7 ± 0.2	1.6		−8.90 ± 0.7			5.0 ± 0.30

* Acceleration $\dot{\omega}_{T_M}$ due to lunar tides only and estimated directly from $\dot{n}_M$.

† Total tidal acceleration ($\dot{\omega}_{T_M} + \dot{\omega}_{T_S}$) consisting of the acceleration described in the first footnote plus the additional tidal contributions discussed in the text.

‡ Mean of *Newton*'s [1970, p. 272] two estimates for the epochs 200 B.C. and 1000 A.D.

§ *Newton*'s [1970] value as reinterpreted by Muller and Stephenson.

¶ Quoted in *Munk and MacDonald* [1960].

components. The major contribution comes from the secular change in a_M or n_M due to the lunar tide M_2 with smaller contributions from N_2 and L_2. Important contributions to $\dot{\omega}_{T_M}$ come from the secular change in e_M (principally due to N_2) and from the secular change in i_M (essentially K_1).

The acceleration due to the solar torque is

$$\dot{\omega}_{T_S} = \frac{1}{C}\frac{Mm_S}{M + m_S} n_S a_S^2\left(\tfrac{1}{3}\cos i_S \frac{\dot{n}_S}{n} + e_S \cos i_S \dot{e}_S + \sin i_S \frac{di_S}{dt}\right) \quad (5b)$$

and although $\dot{n}_S$ will be very small, $\dot{\omega}_{T_S}$ will be important because of the a_S^2 term occurring in (5*b*). The perturbations $\dot{a}_S$, $\dot{e}_S$, and di_S/dt are obtained from (3) by considering the earth as the satellite moving around the sun as planet and computing the changes in the earth orbit due to the tide raised on the earth (as satellite) by the sun (as planet). The ratio of the torques due to the M_2 and S_2 ocean tides then becomes, using the results in Table 2,

$$\frac{(L)_{M_2}}{(L)_{S_2}} = \frac{(\dot{\omega}_T)_{M_2}}{(\dot{\omega}_T)_{S_2}} = \frac{m_M}{m_S}\left(\frac{a_S}{a_M}\right)^3 \cdot \left(\frac{1 + \frac{3}{4}e_M^2 \cos i_M + \frac{3}{2}[(1 - \cos i_M)/\cos i_M]}{1 + \frac{3}{4}e_S^2 \cos i_S + \frac{3}{2}[(1 - \cos i_S)/\cos i_S]}\right) \cdot \frac{(C_{22}^+ \cos \epsilon_{22}^+)_{M_2}}{(C_{22}^+ \cos \epsilon_{22}^+)_{S_2}}.$$

By means of the coefficients given in Table 3 for the two semidiurnal tide models of *Bogdanov and Magarik* [1967] and the error estimates discussed below, this ratio becomes (4.9 ± 1.3). This value is in agreement with the value of 5.1 given by the equilibrium theory [*Munk and MacDonald*, 1960]. *Jeffreys'* [1962] estimate, modified by *Newton* [1968], is 3.8 when the argument of nonlinear friction in the oceans is used. The effect of the solar tides on the $\dot{\omega}_T$ is given in Table 4. For P_1 (*lmpq* = 2100), for which no global ocean data are available, we assume that its contribution is proportional to that of O_1 according to an equilibrium response. That is,

$$(\dot{\omega}_T)_{P_1} = (\dot{\omega}_T)_{O_1}/5.1$$

and similarly for the R_2 tide (*lmpq* = 2201),

$$(\dot{\omega}_T)_{R_2} = (\dot{\omega}_T)_{N_2}/5.1$$

We must also consider the atmospheric tide, which, because of its thermal origin, leads the sun and tends to oppose the ocean tide effects [*Holmberg*, 1952]. Table 3 gives the relevant coefficients, expressed in centimeters of water on the earth's surface, estimated from the model of W. Kertz, B. Haurwitz, and A. Cowley as given by *Chapman and Lindzen* [1970]. The acceleration of the earth will be

$$(\dot{\omega}_T)_{S_2}^{\text{atmosphere}} = \frac{(C_{22}^+ \cos \epsilon_{22}^+)_{S_2}^{\text{atmosphere}}}{(C_{22}^+ \cos \epsilon_{22}^+)_{S_2}^{\text{ocean}}} (\dot{\omega}_T)_{S_2}^{\text{ocean}}$$

Since a major part of the tidal acceleration of the earth can be deduced from the lunar acceleration, the total acceleration can be computed in two ways: (1) $\dot{\omega}_{T_M|a}$ plus the computed contributions due to the tides not contributing to $\dot{n}_M$ (S_2 and P_1), the contributions arising from the secular changes in e and di/dt (both lunar and solar), and the atmospheric tide S_2 and (2) as the sum of all tidal components. The total purely tidal estimate is $(-8.9 \pm 0.6) \times 10^{-22}$ rad s^{-2} as compared with the mean value $(-10.1 \pm 1.0) \times 10^{-22}$ rad s^{-2} from the three astronomically based solutions discussed above and in Table 6. The uncertainty of the $\dot{\omega}_T$ has been estimated from the following uncertainty estimates of the $C_{22}^+ \cos \epsilon_{22}^+$: 10% for M_2; 20% for N_2, R_2, K_1, P_1, and O_1; 15% for S_2; and 10% for the atmospheric tide. The mean of the above two estimates of $\dot{\omega}_T$ corresponds to an average increase in the length of day (lod) of 3.7 ± 0.5 ms/100 yr over the last 3000 years. This value is somewhat higher than the estimate by *Muller and Stephenson* [1975] and represents an improvement for the following reasons:

1. The contribution due to S_2 is based on an actual ocean tide model rather than on any assumptions about the relation between dissipation of tidal energy at the M_2 and S_2 frequencies.
2. Additional solar tides have been considered.
3. Transfer of angular momentum associated with the secular changes in inclination and eccentricity of the lunar and solar orbits has been included.
4. The error introduced by *Newton* [1968] in computing the loading effect of the atmosphere has been corrected (Newton assumed that the load coefficient k_2' was numerically equal to the Love number k_2, whereas $k_2' = -0.30$ and $k_2 = 0.30$).

There remains an uncertainty about the atmospheric tidal contribution in that in the estimates of the C_{st}^+, any response of the ocean surface to the pressure variation has been ignored. If the ocean response is static, an increase in atmospheric pressure will result in a lowering of sea level in the ratio of about 1 cm of water for a pressure increase of 1 mbar over and above the mean pressure over the entire ocean surface [*Munk and MacDonald*, 1960]. Analyses by *Wunsch* [1972] indicate that the response is largely static for pressure fluctuations as short as 40 hours. If at the semidiurnal frequency the response is still largely static, the coefficients in Table 3 will be significantly reduced in amplitude.

Nontidal changes in the earth's rotation. The astronomical observations of the total accelerations of the earth are summarized in Table 6. *Muller and Stephenson* [1975] conclude that *Newton*'s [1970] estimate is in error owing to Newton's assumption that the difference and derivative (ephemeris time minus universal time) vanish at epoch 1900, whereas Muller and Stephenson argue that the derivative vanishes near epoch 1770 [*Munk and MacDonald*, 1960, p. 179]. Their reinterpreted result of *Newton* [1970] is also given in Table 6, and now the agreement is most satisfactory, indicating an increase in the lod of (2.5 ± 0.3) ms/100 yr. Morrison's analysis of the lunar occultations since the seventeenth century gives an increase in the lod of about 1.5 ms/100 yr, but this estimate may reflect not merely the secular acceleration because of the presence of large, long-period, or irregular fluctuations of nontidal origin in the earth's rotation spectrum. Oesterwinter and Cohen have also estimated the variation in the earth's rotation, but their time span of 70 years is too short to give a meaningful estimate of the secular change in lod.

Comparing the astronomical estimates with the computed tidal accelerations indicates the existence of a major secular variation of nontidal origin in the lod. Despite the improved analyses of the astronomical data and the improved estimates of the tidal contributions we are in the same position as Munk and MacDonald in 1960 in that we have to postulate further mechanisms to explain the observed secular change in the earth's rotation. The new magnitudes estimated for these nontidal contributions are larger than those estimated by Munk and MacDonald, and any proposed mechanism must explain a nontidal decrease in the lod of about 1.2 ms/100 yr from the Muller and Stephenson solution, or about 1.6 ms/100 yr from the reinterpreted Newton solution. Of all the acceleration estimates made, the nontidal acceleration is the least satisfactory, since it represents the difference between two quantities each of which is known only to about 10%.

Muller and Stephenson's 'best' estimate for the observed $\dot{\omega}$ gives a nontidal decrease in length of day of 1.3 ± 0.4 ms/100 yr. This nontidal change could be due to (1) further torques acting on the earth's mantle, (2) changes in the earth's inertia tensor, or (3) a decrease in the gravitational constant G. *Rochester* [1973] gives a short review of some of the possible contributions. *Dicke* [1969] assumes that part of the nontidal part of lod is due to $\dot{G}$ and attributes the remaining part to the recent deglaciation and its associated isostatic adjustments. *O'Connell* [1971] attributes all of the nontidal lod to the Pleistocene glaciation and its consequences and arrives at a relation between the relaxation time of the isostatic adjustment of the earth due to a load harmonic in degree 2 and the nontidal change in lod. This relation has two roots, and he estimates that the relaxation time is either about 2000 years or about 100,000 years using the nontidal changes in $\dot{\omega}$ that range between the limits 0.8×10^{-22} and 2.1×10^{-22} rad s^{-2}. These limits are based on the discussion by *Munk and MacDonald* [1960] and on the analysis by *Curott* [1966] of ancient solar eclipses. The above estimate of $(3.4 \pm 1.2) \times 10^{-22}$ rad s^{-2}, based on Muller and Stephenson's discussion of the eclipse data, gives two roots for the relaxation time, one near 4000 years and the other near 40,000 years, although considerable dispersion exists between the different models by O'Connell. These decay times do not differ sensibly from O'Connell's values. Of the two roots, O'Connell prefers the smaller one, and from his discussion it appears that a relaxation time of 4000 years is in agreement with a uniform mantle viscosity of about 10^{22} P or a model in which there would be only a very slight increase in the viscosity below the asthenosphere. However, O'Connell's results may be vitiated by his assumption of exponential decay of the harmonic deformation coefficients, as *Peltier* [1974] argues that the decay is strongly nonexponential.

DISSIPATION OF TIDAL ENERGY

Rate of energy dissipation. The actual rate of dissipation of tidal energy can be computed in several ways, for example, by the time average of the rate of work by the moon on the earth or inversely by the rate of work by the earth on the moon. In view of the fact that $\dot{\omega}$ and $\dot{n}$ have already been computed for the various tide models, the most direct way is to consider the total energy balance of the earth and moon. That is, the rotational energy lost by the earth is

$$\dot{E}_1 = d/dt\ (\tfrac{1}{2}C\omega^2) = C\omega\dot{\omega}_T$$

where both lunar and solar tides contribute to the energy lost (see Table 4 or the astronomically based estimate given in Table 6). The orbital energy gained by the moon is

$$\dot{E}_2 = \frac{d}{dt}\left(\tfrac{1}{2}a_M{}^2 n_M{}^2 \frac{m_M M}{m_M + M} - \frac{GMm_M}{a_M}\right)$$

$$= \frac{d}{dt}\left(-m_M \frac{GM}{2a_M}\right) = -\frac{m_M}{3} n_M a_M{}^2 \dot{n}_M = -Cn_M\dot{\omega}_{T_M}$$

The energy transferred to the earth orbit is only about 1% of that transferred to the moon orbit. When it is assumed that any dissipation in the moon is small, the amount of dissipation in the earth must be

$$\dot{E} = \dot{E}_1 + \dot{E}_2 = C(\omega\dot{\omega}_T - n_M\dot{\omega}_{T_M})$$

From Table 4, $\dot{\omega}_{T_M} \approx 0.8\dot{\omega}_T$. Also, $n_M \approx 0.04\omega$, and $\dot{E} = 0.96C\omega\dot{\omega}_T = 5.6 \times 10^{40}\dot{\omega}_T$ ergs s^{-1}. With the essentially astronomical estimate of $\dot{\omega}_T$ the total rate at which energy must be dissipated in the earth is 5.7×10^{19} ergs s^{-1}. The tidal estimate of $\dot{\omega}_T$ (Table 4) results in a total dissipation in the oceans of 5.0×10^{19} ergs s^{-1}. The difference of 0.7×10^{19} ergs s^{-1} could be interpreted as a source of thermal energy in the mantle, but this value is hardly significant in view of the uncertainties associated with both the tidal and the astronomical estimates of $\dot{E}$. The agreement between these two estimates again stresses the dominant role of the global oceans in dissipating the tidal energy, and any dissipation in the mantle or in the core must be small in comparison. The energy dissipated in the M_2 tide is

$$\dot{E} = C(\dot{\omega}_T)_{M_2}(\omega - n_M)$$

$$= C\frac{m_M}{M}\left(\frac{a_M}{R}\right)^2(\omega - n_M)(\dot{n}_M)_{M_2}$$

$$= -\tfrac{3}{2}C\frac{m_M}{M}\left(\frac{a_M}{R}\right)^2(\omega - n_M)\frac{n_M}{a_M}(\dot{a}_M)_{M_2}$$

$$= -(9/10)\,gm_M\left(\frac{R}{a_M}\right)^3(1 + k_2')(\omega - n_M)\frac{\rho\omega}{\bar{\rho}}$$

$$\cdot(1 + \cos i_M)^2(1 - \tfrac{5}{2}e_M{}^2)(C_{22}{}^+ \cos \epsilon_{22}{}^+)_{M_2}$$

and for the mean of the three models for the M_2 tide the rate of dissipation is 3.7×10^{19} ergs s^{-1}. *Hendershott* [1972] found a comparable 3.04×10^{19} ergs s^{-1} by computing the rate of work done on the ocean by both the tide-generating potential and the sea floor motion due to the solid tide. Figure 1 illustrates the part of the M_2 tide that is represented by the coefficients $C_{22}{}^+$ and $\epsilon_{22}{}^+$. This distribution does not identify the actual energy sinks. Calculations of the dissipation by *Miller* [1966] using the energy flux method give a total dissipation of 1.7×10^{19} ergs s^{-1} and identify some five locations where, together, half this amount is dissipated. It is not obvious how these five regions (the Bering Sea, the Okhotz Sea, Australian to the lesser Sunda Islands, the Patagonia shelf, and the Hudson Strait) relate to the distribution of Figure 1, in particular, since four of the five areas occur in high latitudes, where the tidal torque of the truncated M_2 tide is a minimum. Apparently, any dissipation in a limited area will result in a phase lag over much of

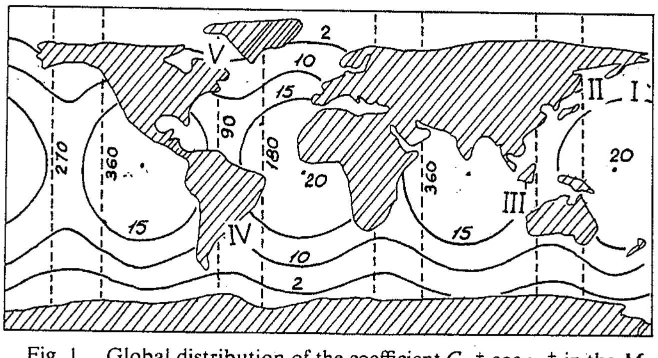

Fig. 1. Global distribution of the coefficient $C_{22}{}^+ \cos \epsilon_{22}{}^+$ in the M_2 ocean tide. The amplitudes (solid lines) are in centimeters. The phase is indicated by the dashed lines. The five principal regions of friction in shallow seas as estimated by *Miller* [1966] are as follows: I, Bering Sea; II, Okhotz Sea; III, Australia to the lesser Sunda Islands; IV, Patagonia shelf; and V, Hudson Strait.

the equatorial regions if, indeed, dissipation is limited to shallow seas only. *Munk* [1968] does not rule out dissipation other than by friction in the shallow seas and concludes that a significant fraction of the dissipation takes place by way of scattering into internal modes and that friction at the deep-sea boundary may be important (see also the reviews by *Hendershott and Munk* [1970] and *Hendershott* [1973]).

Some consequences of the evolution of the lunar orbit. Because the dissipation occurs principally in the oceans, extrapolation of the lunar orbit into the past becomes an impossible task, since we know that a very major reordering of ocean-continent distribution has occurred during at least the last 200 m.y. Assuming constant dissipation, *MacDonald* [1964] showed that the moon must have been very close to the earth about 1.8 b.y. ago. The more recent astronomical estimates will aggravate the time scale problem even more. One way out of this dilemma without invoking nontidal mechanisms is to remove the major part of the shallow oceans for long time intervals during the earth's past. If we can be sure that dissipation is restricted to the shallow seas, a lowering of sea level to a depth below the present continental shelves could conceivably decrease the dissipation. The coefficients C_{42}^+ and ϵ_{42}^+ also give rise to secular changes in the lunar orbit, but at present they are small because of the $(R/a)^2$ factor, which appears in the ratio of perturbations due to C_{42}^+ and C_{22}^+.

As the moon approaches the earth in the backward extrapolation, these terms could conceivably become sufficiently important to change the pattern of the lunar orbit evolution as viewed by Goldreich and MacDonald. Of particular interest are possible changes in the semimajor axis and the inclination. From the ocean potential (2) we can compute the secular rates in these two elements due to both $C_{22}^+ \cos \epsilon_{22}^+$ and $C_{42}^+ \cos \epsilon_{42}^+$ of the M_2 tide. The ratios of the perturbations are, if it is assumed that the orbit remains circular throughout,

$$X_{22/42} = \frac{\dot{a}_{2200}}{\dot{a}_{4210}} = \frac{(di/dt)_{2200}}{(di/dt)_{4210}} = \frac{9}{5}\frac{1+k_2'}{1+k_4'}\left(\frac{a}{R}\right)^2$$
$$\cdot\frac{F_{220}(i)C_{22}^+ \cos \epsilon_{22}^+}{F_{421}(i)C_{42}^+ \cos \epsilon_{42}^+} \approx 0.58\left(\frac{a}{R}\right)^2$$
$$\cdot\left(\frac{7\sin^2 i \cos i}{1+\cos i} - 1\right)^{-1}\frac{C_{22}^+ \cos \epsilon_{22}^+}{C_{42}^+ \cos \epsilon_{42}^+}$$

For an order of magnitude estimation we use the variation in inclination with decreasing semimajor axis as given by Figure 8 of *Goldreich* [1966]. The present ratio $(C_{22}^+ \cos \epsilon_{22}^+)/(C_{42}^+ \cos \epsilon_{42}^+)$ for the M_2 tide is about -3 [*Lambeck et al.*, 1974, Table 2]. We use -1 for this ratio in evaluating $X_{22/42}$. An approximate integration of the influence of the $C_{42}^+ \cos \epsilon_{42}^+$ indicates that the inclination of the lunar orbit would be at most a few degrees larger than that found by Goldreich and only then when the moon was closer to the earth than about 6 R_E. As the inclination function $F_{421}(i)$ changes sign near 30° and again near 80°, the $C_{22}^+ \cos \epsilon_{22}^+$ and $C_{42}^+ \cos \epsilon_{42}^+$ tend to oppose each other between these two values of the inclination. This suggests a possible mechanism for maintaining the moon at constant distance from the earth. But for this to be feasible, $C_{42}^+ \cos \epsilon_{42}^+$ has to be at least 10 times greater than $C_{22}^+ \cos \epsilon_{22}^+$ and only then when the moon is dangerously close to the Roche limit of 2.9 R_E; this is an improbable situation.

Earth's specific dissipation function. If the tidal acceleration is attributed to a solid tide of potential (1) the astronomically deduced value $\dot{a}_M$ can be compared directly with the expressions for $\dot{a}_M$ in Table 1. Assuming that the phase lags are independent of frequency then gives $\epsilon_{22} = 7.5°$, or a Q of less than 10. After correcting the observed $\dot{a}_M$ for the ocean tide there remains a small unexplained change in the lunar orbit of about 0.12×10^{-7} cm s^{-1} if we adopt the mean of the three recent astronomical estimates for $\dot{a}_M$. In view of the uncertainties in both the observed and the computed estimates this difference is probably not very significant, but we can use it to obtain bounds on the possible Q for the solid earth and moon. For dissipation in the moon with Q, $\delta a|_{Q_M} \approx 8 \times 10^{-7} \sin \epsilon_M$, and for dissipation in the earth with Q_E, $\delta a|_{Q_E} \approx 1.2 \times 10^{-6} \sin \epsilon_E$. Then

$$\delta a|_{Q_M} + \delta a|_{Q_E} = (8 \times 10^{-7} \sin \epsilon_M + 1.22 \times 10^{-6} \sin \epsilon_E)$$
$$= (0.20 \pm 0.17) \times 10^{-7}$$

If we assume that the Q for the two bodies are the same, this yields $Q_E = Q_M = 100$, with a lower limit of about 60. This is almost as unsatisfactory as the estimate obtained by *Lambeck et al.* [1974] from analyses of orbits of close earth satellites. A major uncertainty in $\dot{a}_M$ is due to the astronomical observations. For an average mantle Q of 200 the dissipation in the mantle would be of the order of 1.8×10^{18} ergs s^{-1}. The dissipation estimated from the tidal models and that estimated from the astronomical observations are both probably correct to within 10 or 15%, and this places an upper limit on the dissipation of tidal energy in the core of a few units of 10^{18} ergs s^{-1}. This presents a maximum limit on dissipative coupling between the core and the mantle and is an order of magnitude smaller than that used by *Stacey* [1973] in his study of the precessional coupling of the earth's core. H. G. Rochester (private communication) independently argues that Stacey's estimate is at least 2 orders of magnitude too large. More precise limits to the above parameters can be established only with improved ocean tide models and, more important, with improved observational values for the lag angles. Astronomical observations will not provide the latter because of long-period, nontidal variations in rotation. The analysis of the tidal evolution of the orbits of close earth satellites is a more promising, though not yet adequate, method.

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