INFERENCES ON THE LUNAR TEMPERATURE FROM GRAVITY, STRESS STATE AND FLOW LAWS

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Inferences on the lunar temperature regime are made from the inversion of gravity for density anomalies and the stress-state of the Moon's interior, and by comparing these results with flow laws and estimates of likely strain-rates.

The nature of the spectrum of the lunar gravitational potential indicates that the density anomalies giving rise to the potential are mainly of near-surface origin. The average stress-differences in the lunar mantle required to support these density anomalies are of the order of a few tens of bars and have persisted for more than $3 \times 10^9$ years. If current flow laws for dry olivine can be extrapolated to the conditions of the lunar mantle, and the selenotherms based on electrical conductivity models are valid, the strain rates are too high to explain the preservation of the lateral near-surface density anomalies. We suggest that the present temperatures in the Moon are relatively low, of the order of 800°C or less, at a depth of about 300 km. This compares with 1100°C based on electrical conductivity models and is near the lower limit predicted by Keihm and Langseth (1977) from lunar heat-flow observations.

1. Introduction

The various lunar missions have collected a wealth of new data that have led to a much increased understanding of many aspects of the Moon's surface, and has stimulated much discussion on the formation and evolution of not only this body but also of the Earth and other terrestrial planets. However, the data are predominantly surface observations covering limited areas on the lunar nearside, and this makes it impossible to answer uniquely many questions concerning the global properties of the Moon. Thus there is no agreement on the temperature profile of the interior which means that the viscosity and creep laws of the interior cannot be evaluated. This leaves unanswered questions concerning convection in the lunar interior and the mechanisms supporting the lunar topography and lateral density anomalies.

In this paper we first discuss some of the statistical aspects of the lunar gravitational potential, topography and its isostatic support, and the associated stress-state in the interior. Next we have attempted to establish further constraints on the lunar temperature profile from the stress-state and from creep laws. Inferences on temperature remain uncertain because of the ambiguous and limited nature of the observational data. The main source of information are derived from (a) two heat flow measurements, (b) from estimates of the electrical conductivity, and (c) from thermal evolution calculations. None of these methods provide a unique measure of the temperature and all have their built-in biases. Furthermore, results of any one approach appear to have been influenced by results obtained by the other approaches.

Here we have attempted to invert the lunar gravity data for density anomalies and the stress-differences required to support these anomalies, and together with an estimate of strain-rates deduced from the topographic spectrum and with flow laws, deduced likely temperatures in the interior. We do not wish to infer that this approach is free from untestable assumptions than are the other approaches mentioned above, and for this reason we have not attempted a formal inversion of the rheological flow laws. Nevertheless,
our results suggest that the lunar interior may be much cooler than suggested by many of the recent models. Specifically, we find that the temperatures are unlikely to exceed about 0.7 of the melting point temperature for much of the outer 500–800 km of the Moon. If they did, the stresses associated with the density anomalies would have relaxed and the mass concentrations over the ringed maria would not have persisted for $3 \cdot 10^9$ years or longer.

In passing we confirm some older conclusions, namely that: (a) lunar gravity has a predominantly near surface origin; (b) isostatic compensation of the topography is probably not complete everywhere; (c) maximum shear stresses in the mantle due to the topographic load and to density anomalies in the crust and upper mantle do not exceed a few tens of bars; and (d) the second-degree harmonics of the potential are anomalously large relative to the higher degree harmonics, and may be indicative of a fossil bulge. This last point is discussed separately in a second paper (Lambeck and Pullan, 1980).

2. The lunar gravity field

Throughout this paper we are mainly concerned with the relatively long-wavelength features of the lunar gravity field, and this makes it convenient to expand the potential in a spherical harmonic series. Thus, at selenocentric distance $r$, latitude $\phi$ and longitude $\lambda$:

$$U = (1 + \Delta U)GM/r$$

where the non-dimensional anomalous potential is:

$$\Delta U = \sum_{l=2}^{\infty} \sum_{m=0}^{l} \Delta U_{lm}$$

with:

$$\Delta U_{lm} = \left( \frac{R}{r} \right)^{l} (C_{lm} \cos m\lambda + S_{lm} \sin m\lambda)P_{lm}(\sin \phi)$$

In these expressions $G$ is the gravitational constant, $M$ the mass and $R$ the mean radius of the Moon. The $C_{lm}$ and $S_{lm}$ are the fully normalized Stokes coefficients which are related to the lateral density heterogeneities.

Numerous estimates of these Stokes coefficients have been made in the years following the launch of the first lunar orbiters (see Sjogren, 1977, for a brief review), but the results remain unsatisfactory because of a number of well-understood problems. These problems include (i) the absence of far-side tracking data, (ii) the slow convergence of the lunar potential coefficients, (iii) the inadequate distribution of the inclinations of low perigee satellites, and (iv) a lack of long arcs of tracking data that are unperturbed by spacecraft manoeuvres. Different analysis techniques have been developed to offset some of these limitations, mainly by directly mapping the spacecraft's observed line-of-sight accelerations and by representing the anomalous density structure of the Moon by point or disc masses at specified locations on, or below the surface (e.g. Muller and Sjogren, 1968). But these methods give information only for the near side of the Moon and do not yield a global solution.

The low degree terms of the spherical harmonic expansion of the lunar gravity field have been estimated either directly, by expressing the spacecraft velocities or rates of change of the orbital elements as functions of the potential coefficients, or indirectly, by first estimating mass points and then relating these to the spherical harmonic coefficients. Two recent solutions are by Ananda (1977) and Ferrari (1977). Some very low degree harmonic coefficients, $l = 2$ and 3, have also been determined from the lunar librations using laser range observations (King et al., 1976; Williams, 1977). The situation is unsatisfactory for the harmonics of degree 4 and higher. Even for $l = 2$ the uncertainties in the present solutions lead to an uncertainty in the moment of inertia of about 1%, and this is too large to permit definitive conclusions to be drawn about the nature of the lunar core (see e.g. Ringwood, 1979). For the higher degree harmonics the differences between the various solutions often exceed the values themselves. The solutions are illustrated in Fig. 1 in the form of the power spectra:

$$V_l(U) = \sum_{m} (C_{lm} + S_{lm})$$

(2)

The spectrum based on Ferrari's (1977) solution gives power estimates that are systematically greater than those based on Ananda's (1977) solution. Also illustrated in Fig. 1 is the spectrum of the differences between the two solutions. Figure 2 illustrates the correlation, or rather the lack of correlation, between
Fig. 1. Gravitational potential power spectra based on solutions by Ananda (1977) (a), Ferrari (1977) (b), the difference spectra (c) of these two solutions, Michael et al. (1969) (d) and Michael and Blackshear (1972) (e).

the two solutions. Together, these two figures give us little confidence in the potential coefficients of the presently available expansions. Ananda's (1977) solutions leads to a somewhat better correlation between gravity and the topography of Bills and Ferrari (1977) than does Ferrari's solution (Fig. 2), indicating that the former solution may be preferable. Also illustrated in Fig. 1 are solutions by Michael et al. (1969) and Michael and Blackshear (1972) which have been used in several recent studies of the stress-state of the Moon (e.g. Arkani-Hamed, 1973a, b; Kuckes, 1977). These solutions give highly unrealistic gravity anomalies for the lunar farside and the models are not significant beyond about degree 4.

Figure 3 illustrates what we consider to be the best estimate of the spectrum. For $l = 2-4$, it is based on the average of the independent solutions by Michael et al. (1969), Lui and Laing (1971) and Sjogren (1971), as well as on the libration study by King et al. (1976) and Williams (1977). The spectrum for $l > 4$ is based on a smoothed version of the average of the Ananda (1977) and Ferrari (1977) solutions. The spectrum of the perturbation in gravity at $r > R$ is

\[ V_l^2(\Delta g) = g_k^2(l + 1)(R/r)^{2l+6} V_l^2(\Delta U) \]

where $g_k$ is gravity at the lunar surface $r = R$. The total power for $l = 3-18$, corresponding to $10^5 \times 10^6$ areas, at 100 km altitude is of the order 44 mgal$^2$ for the smoothed composite spectrum given in Fig. 3. This can be compared with Kaula's (1969) estimate of the mean square gravity perturbation values for $10^5 \times 10^6$ areas of 33 mgal$^2$ for the lunar near-side at
100 km altitude, based on the Muller and Sjögren (1968) model. Gottlieb (1970) concludes that the line-of-sight accelerations may be underestimated by about 30% and the agreement is therefore satisfactory. Ferrari’s (1977) solution for \( l = 3 - 16 \) yields 69 mgal

The spectrum of the Earth’s gravitational potential adheres closely to a power law:

\[
V^2(\Delta U) = A (2l + 1) \times 10^{-10} l^{-4}
\]

which can be attributed to density anomalies, randomly distributed in depth as well as laterally (Lambeck, 1976). Most of the terrestrial gravity field originates from density anomalies in the upper mantle, but contributions from the lower mantle are also required to explain the low-degree spectral estimates, for without them the decay would be less rapid than the above law (see also Kaula, 1977). This terrestrial spectrum decays more rapidly than that of the Moon (Fig. 3) which suggests that, for the latter, the density anomalies are more superficial. This is not to say that density anomalies do not occur in the upper mantle but, if they do, they must be proportionally smaller, otherwise the potential power spectrum would decay more rapidly than is observed. However, while a near-surface distribution can explain the spectrum for \( l \geq 3 \), it cannot explain the spectral estimate for \( l = 2 \) in its entirety. If deeper density anomalies are introduced to satisfy the \( l = 2 \) estimate, the rest of the spectrum will decay more rapidly than it appears to do. No simple distribution of randomly distributed density anomalies can explain the entire spectrum, and the second-degree spectral estimates clearly lie above some regular decay rule.

3. The lunar topography

The lunar topography, deduced from a combination of Earth and lunar orbiter photography and radar measurements, has been harmonically analysed by Bills and Ferrari (1977), and can be expressed by the expansion:

\[
h = R \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left( \bar{h}_{lm} \cos m \lambda + \tilde{h}_{lm} \sin m \lambda \right) \bar{P}_{lm}(\sin \phi)
\]

The spectrum of this topography, referred to a sphere of radius \( R \) is given by an expression analogous to eq. 2. It is illustrated in Fig. 4. Bills and Ferrari (1977) computed mean values for 5° \( \times \) 5° areas where observations of elevation were available and predicted mean values for the unsurveyed areas using linear-regression techniques. This could lead to some uncertainty in the topography spectrum since they assumed that the statistical distribution of the heights can be characterized by a stationary process. An analysis of the lunar topography by Chuikova (1976), based on partly independent data, gives the second spectrum illustrated in Fig. 4. Agreement between the two spectra is satisfactory up to degree six, but beyond that the Bills and Ferrari (1977) spectrum gives consistently higher spectral estimates. Bills (1978) also carried out some Fourier analyses of the lunar topography along great circles and found that the spectrum decayed approximately by \( 1/l^2 \). This is similar to the decay of the Earth’s topographic spectrum (Vening-Meinesz, 1951; Balmino et al. 1973). Bills (1978) suggests that the lunar spectrum decays approximately according to the expression:

\[
V^2(h) = 1.5 \cdot 10^{-6}/l(l + 1)
\]

although his observed spectrum exceeds this for \( l > 8 \). For much of the subsequent discussion we adopt this spectrum, also illustrated in Fig. 4.

An outstanding feature of the topography spectrum is that the power in the \( l = 1 \) harmonics is considerably greater than that in the higher-degree harmonics. The \( l = 1 \) harmonics indicate that there is an offset between the center of mass and the center of figure, with the center of mass being closer to the Earth by about 2.0 km (Bills and Ferrari, 1977). For Mars the offset is about 2.5 km and for the Earth the offset is about 1.1 km. There would be nothing special about the lunar result were it not for the fact that, for the Moon, this spectral estimate significantly exceeds the \( l > 1 \) estimates, whereas for the Earth all low-degree \( (l < 6) \) spectral estimates are of comparable magnitude and for Mars \( V^2(h) \) exceeds the power at \( l = 2 \) and 3 only by a factor of 2 (Fig. 4).

In so far as the lunar offset is roughly along the Earth–Moon axis, it is tempting to associate it with the dichotomy seen in the lunar surface of a nearside dominated by maria and farside dominated by highlands. Several suggestions have been made. Kaula et
al. (1974) suggested that there may be difference in the crustal thickness between the near- and farsides, the highlands being associated with a thicker crust than the maria. Alternatively, adopting a Pratt-type crustal model, the crustal densities of the highlands would be less than those of the mare regions. Ransford and Sjogren (1972) have suggested that the core is offset, while Lingenfelter and Schubert (1973) suggested that the offset may be a result of a first-degree convection cell in the mantle. All these explanations consider that the offset was formed at an early stage in the Moon's history and that it has been "frozen" into the planet ever since. Thus the models have to be tested not only against the plausibility of the mechanism causing the offset, but also against the consequences of this offset on the stress state of the planet.

The usual theory of the loading of a homogeneous incompressible sphere, the so-called Kelvin body (Jeffreys, 1970, Appendix B), is not directly applicable for an $l = 1$ load since, in this case, one solution of the elastic equations of motion is a constant but indeterminate shift of all particles of the body and involves no stresses. The solution is discussed by Farrell (1972). Table I summarizes the total strain energy which may be expressed:

$$E_l = \frac{3}{2(2l + 1)(\rho_0 p)} \int_0^h \bar{\nabla}(h)\bar{E}_l$$

for a homogeneous compressible Moon loaded by the topography whose spectrum is defined by eq. 4. $E_l$ is the total strain energy of the planet due to the deformation under a surface load of unit gravitational potential. Of note is that the strain energy associated with the large $l = 1$ harmonics in the topography is negligible compared to that associated with the higher degree, smaller amplitude, harmonics. One interpretation is that during the final stages of the lunar surface bombardment, the overall topography was much greater than it is now, with a spectrum whose power for low degrees approximated that in the present $l = 1$ harmonics. This would be the case if the characteristic wavelength of the process responsible for the initial topography were much shorter than the wavelengths corresponding to the low degree wavenumbers $l$. The early topography, presumably in a state of near-isostasy, would have resulted in stresses in the crust and upper mantle that were an order of magnitude larger than those existing for the present topography and any subsequent adjustment would have

### Table I

<table>
<thead>
<tr>
<th>$l$</th>
<th>$E_l$ ($10^6$ ergs)</th>
<th>$(\bar{T})$ (bar)</th>
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<tr>
<td>12</td>
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<td>0.5</td>
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Column 2 is for the topography of Bills and Ferrali (1977) and column 3 is for the topography given by eq. 4. Column 4 gives the maximum stress differences in the Moon due to the topography of Bills and Ferrali (1977) as given by the approximate rule (8).
been one from this isostatic state to a more nearly hydrostatic equilibrium state.

A principal conclusion arising from the discussion in Section 2 on the lunar gravity field is that the observed potential originates almost entirely from density anomalies in the lunar crust. Despite this near-surface origin, there is little obvious global correlation with elevation differences. This need not imply that the lunar crust is in a state of isostatic compensation since, if it were, free-air gravity anomalies would correlate positively with elevation. Rather, it may reflect the unsatisfactory nature of the present global gravity and topography solutions, or it may be due to density anomalies in the crust that do not have a major topographic expression. It could also be a consequence of a dichotomy of processes forming the lunar crustal features. Normally, highlands will be associated with positive gravity since elevated regions are closer to the heights at which the potential is sensed than to any compensating and deeper "roots". For the same reason the lower elevation regions should give negative anomalies, were it not for the presence of the abnormal mass concentrations found to underlie all major circular maria. Any further study of these correlations would have to be done on a regional basis rather than globally.

4. Isostasy and mantle stresses

If the lunar topography is entirely uncompensated, then the expected gravitational potential spectrum could be expressed as:

$$V_l^2(\Delta l' ) \sim \left( \frac{3\rho_c}{\bar{\rho}} \right)^2 \frac{1}{(2l+1)^2} V_l^2(h)$$

while if this topography is fully compensated, and the compensation can be described by an Airy-model, then:

$$V_l^2(\Delta l' ) \sim \left( \frac{3\rho_c}{\bar{\rho}} \right)^2 \frac{1}{(2l+1)^2} \left[ 1 - \left( \frac{R - D}{R} \right)^2 \right]^2 V_l^2(h)$$

where $\bar{\rho}$ denotes the mean density of the planet. Utilizing eq. 4 the first spectrum decays with increasing $l$ according to $l^{-4}$ and the second spectrum decays according to $l^{-2}$. These two spectra are compared with the observed spectrum in Fig. 5. As for the Earth and Mars (Lambeck and Pullan, 1980), the spectrum expressed by eq. 5 exceeds the observed spectrum for all $l$, indicating that some compensation of the topography occurs. For a compensation depth of 50 km the spectrum expressed in eq. 6 is less than the observed spectrum. This suggests that either the compensation is incomplete or that the depth of compensation is greater than 50 km. With $D = 100$ km, eq. 6 gives a spectrum of similar magnitude to that determined by Ananda (1977) for all values of $l > 2$.

Seismic data indicate that the crust in the Oceanus Procellarum region may be of the order of 65 km thick (Töksoz et al., 1974). As the highlands are on the average about 3 km higher than the maria, the expected crustal thickness under the former is of the order of 80 km. Some evidence for a variation in crustal thickness is found in the Apollo 16 seismic data for the Descartes region of the central highlands. Here the crust is estimated to be some 15 km thicker than under the Procellarum region (Goins et al., 1977), while the average elevation of the Descartes region is about 2 km above Oceanus Procellarum.

![Fig. 5. Gravitational potential spectra for the Moon based on:](image)
(Sjogren and Wollenhaupt, 1973). These seismic estimates are less than the 100 km estimated above from the gravity data and isostasy may not be complete, possibly due to the incomplete compensation of the mass concentrations over the circular maria (Kaula, 1975). Certainly the Apennines (Ferrari et al. 1978) and Copernicus (Sjogren et al. 1974) do not appear to be in isostatic adjustment. Alternatively, the isostatic mechanism may differ from the simple Airy model, or Ananda’s (1977) spectrum overestimates the power in the potential.

If the topography loads a homogeneous elastic planet the maximum stress-difference $\tau$ arising in the planet will be of the order $(1/2)p\bar{g}h$, if there are no active tectonic processes acting in the region. The maximum values occur at a depth of about $R/(l - 1)$. Jeffreys (1943, 1970), using the criteria of minimizing stress-differences, suggests that the maximum values occurring in the planet may be somewhat less than this, or:

$$\tau \sim (1/2 \text{ to } 1/3)\bar{g}h$$  \hspace{1cm} (7)

Hence the spectrum of the maximum stress-differences would be of the order:

$$V_1^2(\tau) \sim (1/2p\bar{g}R)^2V_1^2(h)$$  \hspace{1cm} (8)

Table I summarizes the results for the Moon. Of the planetary bodies, the Moon, Mars and the Earth, it is probably the first where Jeffreys’s stress models are most applicable in view of the obvious absence of major recent tectonic processes and the low level of seismic activity; there has been a longer time interval for the equilibrium condition to be approached, and there are no significant active tectonic processes that may accentuate the topography and support it dynamically rather than statically.

More precise estimates of the stress-state can be obtained by solving the equations for an elastic planet, loaded by a harmonic density layer (Longman, 1963; Farrell, 1972). To simplify the calculation we have computed the average distortional strain (shear) energy $E_s$ in a layer of radius $r$ (e.g. Kovach and Anderson, 1967). This was convenient in that $E_s$ can be related to the stress-difference and that a computer program developed by J.B. Merriam was available. The value of $E_s$ is related to $\tau$ through the von Mises function $F$ (Jeffreys, 1970) by the expression:

$$\tau \sim (4/3)^{1/2} = (8/3)^{1/2}\mu E_s$$

$\tau$, as defined here, represents the maximum stress-difference averaged over a shell of radius $r$.

The calculation has been carried out for the several
lunar models illustrated in Fig. 6. Models M2 and M2A are essentially two-layer models, the former having a sharp discontinuity in rigidity at a depth of 1000 km and the latter having a gradual change in rigidity through a 100 km layer centered at a depth of 500 km. Both models are characterized by the known mean-density of the Moon and a constant Lamé parameter $\lambda$. The third model, M3, has elastic properties for the outer 1200 km that are consistent with the seismic-velocity profiles of Goins et al. (1978). Below 1200 km the rigidity decreases linearly through three orders of magnitude. Figure 7 summarizes the results for a normal, white noise, topographic spectrum of $V^2(\eta) = 5 \cdot 10^{-8}$. In model M3, the stresses are quite uniformly distributed with depth while in the two other models the stresses tend to concentrate at the boundary across which the rigidity changes rapidly. Convergence of $\tau_i$ occurs only if the load converges and there is a possibility that the stresses will reach a second maximum near the surface. For M3 the stress-differences closely follow that predicted by the approximate rule of eq. 8 and, as already noted, the maximum stresses for any $l$ occur at a depth of roughly $R/(l - 1)$. This model also closely approximates the response of a homogeneous planet except for $l = 2$, for which the maximum stress-difference occurs at the center. For the other models the stress-differences are also adequately given by the rule expressed by eq. 8, provided that the thickness of the solid layer equals or exceeds $\sim 2R/(l - 1)$.

To evaluate the maximum stress-differences due to a load comprising several harmonics, stresses and strains should be computed individually for each harmonic as a function of latitude and longitude, summed at each point over the contribution from each degree and order and only then can the maximum differences be determined. But to keep the calculations within reasonable limits we use two measures of the stress-differences:

$$\bar{\tau}_1(r) = \left[ \sum_l \tau_l^2(r) \right]^{1/2}$$  \hspace{1cm} (9a)

and:

$$\bar{\tau}_2(r) = \sum_l \tau_l(r)$$  \hspace{1cm} (9b)

The former will represent a measure of the average stress-difference whilst the latter will give an upper limit to the maximum stress-difference as a function of depth.

Figure 8 illustrates the functions $\bar{\tau}_a$ and $\bar{\tau}_b$ for the three Moon models which have been loaded at the surface by a topography whose spectrum is that given by eq. 4. The maximum stress-differences are relatively small, and will not exceed about 60 bar for model M3.

If, as concluded in Section 2, most of the density anomalies in the Moon are concentrated in the lunar crust and uppermost mantle, the stresses in the planet due to these lateral density contrasts can be evaluated with eqs. 9a and 9b by using an equivalent uncompensated topography that gives rise to the observed gravity.
This gives the results illustrated in Fig. 9 using the average gravitational potential spectrum in Fig. 3. These stress-differences are about one half of those due to the actual topography if it were uncompensated.

A more correct solution to computing the stresses associated with the gravity anomalies is to invert the gravity data for both internal density anomalies and

the stress state, using the condition that the overall strain energy is minimized. Such an approach has been used by Kaula (1963) for the Earth and by Arkani-Hamed (1973a, b) for the Moon. In a first calculation by Arkani-Hamed (1973a) the density variations were confined to the upper 50 km of the Moon and the associated stress-differences were found to be of the order of 40 bar within the upper 800 km. The maximum value obtained was 70 bar. As Arkani-Hamed (1973a) used the potential solution by Michael et al. (1969) (which significantly overesti-
mated the spectrum for \( l > 5 \) (Fig. 1), this result is compatible with those given in Fig. 9; the use of Michael et al.'s (1969) model leads to an overestimation of stress-differences at shallow depths by a factor of about four. In a second paper Arkani-Hamed (1973b) permitted the lateral density anomalies to be distributed throughout the Moon and obtained maximum stress-differences of about 100 bar, using the potential model by Michael and Blackshear (1972), which is even less palatable degrees for \( l > 4 \) than the earlier model (see Fig. 1).

Several comments are appropriate at this stage. The stresses given by eqs. 9 and Figs. 8 and 9 are small, a few tens of bars at most, depending on the criteria used for combining the stress estimates for individual harmonics. They represent the stress-differences averaged over the shells and at any point the actual value may be greater than this. On the other hand, the topography is assumed to be uncompensated, and any compensation will reduce the stresses as demonstrated by Jeffreys. The low values are compatible with the lack of tensile or compressive features on the lunar surface and with the low level of seismic energy release; the only tectonic features on the Moon appear to be associated with vertical motions, mainly the down-faulted rilles and lunar scarps, and possibly with the relaxation of the topographic associated stresses around some maria, in particular Mare Orientale.

The low stress-differences required to support the lunar topography also raises the possibility that the topography may not be entirely compensated. This is seen more clearly in studies of some features such as the Montes Apenninus, an old lunar hilly region dated at \( \sim 3.9 \times 10^9 \) yr, which appears to be uncompensated. The shear stresses set up by this feature are of the order of 60 bar and occur at a depth of about 60 km (Ferrari et al., 1978). At a depth of 200 km the maximum stress-difference will still be of the order of 30 bar. These results argue both for a crustal thickness of at least 60 km in mare regions, and for the ability of both the crust and upper mantle to support non-hydrostatic stresses of several tens of bars for as long as \( 4 \cdot 10^9 \) yr.

Kuckes (1977) has estimated the stresses set up in the Moon by the mass concentrations under the circular mare by assuming that these mass concentrations can be considered as loads on a thin elastic shell overlying a plastic substratum. He obtains values of the order of 500–1000 bar for Mare Orientale and argues that this is acceptable when compared with stresses occurring in the Earth's lithosphere due to loading. Kuckes (1977) finds further support for his conclusion in the spectrum of the lunar gravity field fluctuations, but unfortunately the spectrum he used, based on a solution by Michael and Blackshear (1972), is quite unrealistic for values of \( l > 4 \) (see Fig. 1). Moreover, it is improbable that either the Earth's or Moon's crust can support kbar stresses for several billion years (see below). Neither is it evident that the thin plate loading theory adequately describes the lunar mass concentrations.

Jeffreys' approach, of seeking a solution that minimizes the shear stresses may be more relevant. For Mare Imbrium, the mass excess \( \Delta M \) is of the order \( 2 \times 10^{21} \) g, distributed over an area \( A \) of \( 4 \times 10^{15} \) cm\(^2\) (Kaula, 1969). The maximum stress-difference is:

\[
\tau \sim (1/2)g \Delta M/A \sim 40 \text{ bar}
\]

For Serenitatis \( \Delta M \sim 10^{21}, A \sim 2 \times 10^{15} \) cm\(^2\) and \( \tau \) is also \( \sim 40 \) bar. The depths at which these maximum stresses occur are \( \sim 200 \) km for Imbrium and 150 km for Serenitatis. It is noteworthy that these stresses are not very different from those associated with the smaller gravity anomaly over the Apennines. It is also noteworthy that in all these cases significant stress-differences occur down to considerable depths below the crust.

5. Rheological implications

While the lunar crust is relatively thick, on the average about 70 km (see above, and Kaula, 1977), the total stress-differences set up in the mantle by the overlying topography and crustal density anomalies, are still of the order of a few tens of bars down to depths of at least 500 km, and the upper mantle must be sufficiently strong to have supported them for very long time intervals. Seismic models indicate a very high shear \( Q \) upper mantle down to a depth of about 300 km (Nakamura et al., 1976) or about 500 km (Goins et al., 1978), suggesting that the lunar mantle is depleted in water and that there is no partial melting. Nakamura et al.'s (1976) model contains a middle mantle, extending down to a depth of
about 1000 km, in which there is more attenuation but in which \( Q \) is still very high compared with the Earth’s mantle \( Q \). In this region the S-wave velocity gradient is negative. Goins et al. (1978) suggest an important decrease in S-wave velocity at 500 km depth. It has also been suggested that below 1000–1200 km the mantle may be in a state of partial melting. Deep focus moonquakes, which indicate a strong tidal periodicity, have been recorded at depths near 800 km and deeper. This suggests that the deep lunar interior still behaves in a brittle manner when subject to stress differences beyond a critical limit, at least for loads of short duration. Whether this deeper region can support loads over geological time periods becomes uncertain since temperatures there must be reaching a significant fraction of the solidus of lunar materials.

Estimates for the finite strength, or resistance to creep, of the Earth’s crust, range from a few bars to more than 1 kbar. In the present context we are interested in knowing what part of a planet’s gravity field can be supported elastically by the crust and lithosphere for long time intervals. If the upper limit of several kbars for the finite strength is relevant, then much of the Earth’s gravity field can be attributed to density anomalies in the lithosphere (Lambeck, 1972). But if the finite strength is <100 bar, the density anomalies must be dynamically supported (e.g. Kaula, 1972). For the Moon, where the stress-differences are much less than occur in the Earth, such a clear cut distinction is not possible.

Creep of mantle materials subjected to stress-differences is believed to occur by diffusion or dislocation processes (see Nicolas and Poirier, 1976, for an overview). Weertman (1970) argued that dislocation creep is dominant at stresses above \( 10^{-2} \) bars but this value is critically dependent on the adopted grain size. Weertman (1970) adopted a grain size of 1 cm, but for a grain size of 0.1 cm dislocation creep would be dominant only at stresses above \( 10^6 \) bar. Kirby and Raleigh (1973) concluded that dislocation creep is dominant throughout the Earth’s mantle for strain-rates in the range \( 10^{-16} - 10^{-13} s^{-1} \) and that Nabarro-Herring diffusion creep is dominant only at lower rates. Stucker and Ashby (1973) concluded that dislocation creep is dominant for stresses of \( 1 - 500 \) bar but for strain-rates less than \( \sim 10^{-14} s^{-1} \), Coble creep (grain boundary diffusion creep) may be important.

The constitutive equation for creep at stresses below \( \sim 5 \) kbar is:

\[
\dot{\varepsilon} = A\varepsilon^n \exp[-(E^* + PV^*)/RT]
\]

where \( \dot{\varepsilon} \) is the strain-rate, \( E^* \) the activation energy, \( V^* \) the activation volume, \( R \) the gas constant, \( P \) the confining pressure and \( T \) the absolute temperature. The constants \( A \) and \( n \), as well as \( E^* \) and \( V^* \), depend on the chemical and mineralogical composition of the material and on the particular mechanism that dominates the creep process. The value of \( E^* \) may be strongly dependent on the presence of water in the mineral assemblage (Post, 1977). In the Moon \( PV^* \ll E^* \) and the choice of activation volume is not critical. The choice of \( E^* \) is important since a change of 10% will result in a change in strain-rate of about three orders of magnitude. Values for \( n \) of 2–9 have been reported in the literature (e.g. Mercier et al., 1977) but \( n \approx 3 \) appears in order for the stresses considered here. In the low stress and strain-rate regime, a lower value for \( n \) may be appropriate but there is little relevant observational evidence for this. Clearly the choice of \( n \) is even more critical than the choice of \( E^* \).

An important source of uncertainty in evaluating \( \dot{\varepsilon} \) is the temperature profile. Observational evidence for the lunar temperature profiles comes from two indirect sources: electrical conductivity measurements and heat flow observations. The response of the Moon to transient variations in the ambient magnetic field gives estimates of the electrical conductivity of the planet (e.g. Dyal et al., 1976) and comparison of these results with laboratory measurements gives a measure of the selenotherm (e.g. Duba et al., 1976). The inversion of the magnetic data for the electrical conductivity profile is not unique, and further limitations arise from the nature of the observed phenomena and measurements technique. As a result, the conductivity profiles are uncertain and Dyal et al. (1976) suggest that the best results for the electrical conductivity, obtained between depths of 300 and 900 km, give a range of permissible values ranging over half an order of magnitude. Goldstein (1978) however is less sanguine about the ability to determine reliable conductivity results and argues that nothing can be said about conductivity values below about 1000 km depth. The second inversion, from conductivities to temperatures, is perhaps even more
uncertain as this requires a quite detailed knowledge of both the chemistry of the planet and of the conduction mechanism appropriate to the planet's interior. Duba et al. (1976) estimate that at depths of 200–600 km the uncertainty in the temperature is ~150 K (Fig. 10), but the actual situation may be worse than this. Ringwood (1979) argues that these temperatures are too high for two reasons: (a) oxygen fugacities are probably substantially higher than previously thought and this will lower the temperatures, and (b) the presence of Cr$^3+$ and Al$^{3+}$ further reduces the temperatures predicted from electrical conductivity by as much as 200–300 K (Huebner et al., 1978).

The two lunar heat-flow measurements lead to quite uncertain estimates of the global average heat-flow. To deduce the selenotherm requires that further assumptions be made about (a) the distribution of radioactive elements in the crust and mantle, and (b) the thermal conductivity profile. Keihm and Langseth (1977) conclude that at a depth of 300 km, a wide range of temperatures, (1100–1600 K) are compatible with the estimated average heat flow and with differentiation models. Errors in the thermal conductivity model may lead to a further uncertainty of 100–200 K. Keihm and Langseth (1977) appear to favour temperatures near the upper limit of the above range, a choice that seems to be guided by the unrevised interpretation of the electrical conductivity results. Figure 10 summarizes the various results and illustrates our "preferred" selenotherm, which results in lower temperatures at a given depth than do the above mentioned models. This choice also lies further from the pyroxenite solidus of Ringwood and Essene (1970) than the other suggested thermal models, but we suggest that this model is compatible with: (i) the above-discussed indirect observational data; (ii) the notion that the stresses in the lunar mantle have persisted for long time periods; (iii) moonquake activity occurring down to depths of 1000 km indicating that the temperatures depart significantly from the solidus down to at least this depth; and (iv) the argument by Ringwood (1970, 1979) that the mare basalts originated from depths greater than several hundred kilometers. More recently Delano and Ringwood (1979) argued that the depth of the basalt source exceeded 400 km. As these basalts mostly erupted prior to about $3 \cdot 10^9$ y the selenotherm could have approached the solidus only below these depths. The subsequent thermal evolution could not have moved this point of contact upwards for otherwise the stresses associated with the mass concentrations would have relaxed. The Moon's relatively large surface-area to volume ratio suggests instead that considerable internal heat has been lost since the mare formation and that the upper mantle temperatures must now be well below the solidus.

For the typical lunar mantle conditions, an uncertainty of 100 K in the temperature results in a change of the strain-rate according to eq. 10 by nearly two magnitudes. With this model the ratio $T/T_m$, where $T_m$ is the melting point temperature, exceeds 0.5 and power-law creep can be expected with $E^*$ independent of temperature (Nicolas and Poirier, 1976).

Table II summarizes some recent experimental values for $A$, $E^*$ and $n$ for "dry" dunite aggregates and for single olivine crystals (see also Carter, 1976; Goetze, 1978; Paterson, 1979). In view of the absence of volatiles in the lunar crust and interior, the choice of dry parameters appears appropriate. Also, while terrestrial mantles are not composed exclusively of olivine but also contain pyroxenes, garnets etc, the creep strength of these compositions are believed to be similar to that of olivine (Goetze, 1978). The
TABLE II

Creep parameters for olivine

<table>
<thead>
<tr>
<th>Source</th>
<th>( A ) (kbar(^{-n}) s(^{-1}))</th>
<th>( E^* ) (kcal/mole(^{-1}))</th>
<th>( n )</th>
<th>( \dot{\epsilon} ) s(^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carter and Ave'Lallemand (1970)(^a)</td>
<td>5.1 ( \times ) 10(^8)</td>
<td>111</td>
<td>3.3</td>
<td>8.5 ( \times ) 10(^{-19})</td>
</tr>
<tr>
<td>Kirby and Raleigh (1973)</td>
<td>1.8 ( \times ) 10(^8)</td>
<td>100</td>
<td>3.0</td>
<td>1.2 ( \times ) 10(^{-17})</td>
</tr>
<tr>
<td>Kohlstedt and Goetze (1974)(^b)</td>
<td>2.8 ( \times ) 10(^11)</td>
<td>126</td>
<td>3.0</td>
<td>3.5 ( \times ) 10(^{-19})</td>
</tr>
<tr>
<td>Post (1977)</td>
<td>4.3 ( \times ) 10(^8)</td>
<td>126</td>
<td>3.0</td>
<td>5.5 ( \times ) 10(^{-22})</td>
</tr>
<tr>
<td>Durham and Goetze (1977)</td>
<td>6.9 ( \times ) 10(^11)</td>
<td>125</td>
<td>3.6</td>
<td>8.5 ( \times ) 10(^{-20})</td>
</tr>
</tbody>
</table>

\(^a\) As revised by Carter (1976, p. 335). \(^b\) See Carter (1976, p. 335). The last column gives the strain rates for nominal pressure, temperature and stress.

results by Carter and Ave'Lallemand (1970) have been revised by Carter (1976). These results, as well as those of Kirby and Raleigh (1973) and Post (1977) are based on polycrystalline aggregates. The shear stresses used in these tests are of the order of kbar. Kohlstedt and Goetze's (1974) (see Nicolas and Poirier, 1976, p. 181, for revised values) and Durham and Goetze's (1977) values are for dry single olivine crystals. These authors conclude that polycrystalline olivine results fall in these ranges. Kohlstedt and Goetze's (1974) results are for stresses from 50 bar to 10 kbar while Durham and Goetze's (1977) values are for stresses of >200 bar. For a given temperature, the different sets of parameters result in strain rates that differ by more than four orders of magnitude (Table II). The activation volume is set to 13 cm\(^3\) mole\(^{-1}\) (Ross et al., 1979). Post's (1977) results give the lowest strain rates and he argues that this is due to the sensitivity of \( E^* \) to the presence of water. Post's (1977) conclusions seem hard to reconcile with the single crystal data of Durham and Goetze (1977), and Kohlstedt and Goetze (1974). Paterson (1979) concludes that the flow of dry dunite is not well known and that there is no uncontro-roversial evidence that the absence of water increases the creep-strength (see also Goetze, 1978).

Admitting that the extrapolation from these laboratory experiments to the Moon is valid, the strain-rates can be computed and combined with the stress-differences in the Moon due to the anomalous lateral density-variations. Figure 11 summarizes the results for three lunar models discussed previously (see Fig. 6), the corresponding average stress-differences given in Fig. 9, and the creep law parameters given by Durham and Goetze (1977) and by Post (1977).

To make use of these results we require estimates of the strain-rates. On Earth, global geological strain-rates are typically of the order \( 10^{-14} - 10^{-17} \) but they will be considerably less on the Moon. Perhaps an upper limit is obtained from the earlier observation that the present topographic spectrum may indicate a relaxation of stresses. Then, for the low degree harmonics the estimated strain is of the order:

\[ \frac{V_i^2(h) - V_i^2(0)}{h^{1/2}} \approx 2 \cdot 10^{-3} \]

(with \( l = 2, 3, 4 \) in say, \( 3 \cdot 10^9 \) y, and the strain-rates are \( \sim 10^{-19} - 10^{-21} \) s\(^{-1}\).)

If the anomalous mass distributions have existed for \( 3 \cdot 10^9 \) y longer, the flow laws must result in strain-rates that lie below the above limits throughout that part of the mantle that, in the stress calculation, has been assumed to be capable of supporting any stress at all. For the lunar model M2A, for example, this layer extends from the surface down to 600 km.
6. Discussion

The results suggest a number of possible situations for the lunar interior:

(i) Stresses are confined to the upper-most part of the mantle. Maximum values will be increased but, for a given selenotherm, the strain-rates may be lower because of the relatively lower temperatures in the stress-bearing layer. This is illustrated in Fig. 12 where the strain rates have been computed for the models previously discussed as well as for two models with thinner lithospheres; model M4 is for a 400 km thick lithosphere and model M5 is for a 300 km thick layer. But a simple trade-off is not possible without further information since a thinner lithosphere will also imply higher temperatures in the lunar interior.

(ii) The adopted selenotherm (Fig. 10) still results in temperatures that are too high in the lunar interior. With Post's (1977) rheology, temperatures about 100 K lower at 400–600 km result in acceptable strain-rates for the model M2A (Fig. 12). In view of the renewed doubts expressed about the knowledge of the selenotherm these lower temperatures may not be unreasonable. For model M2, the temperatures have to be decreased by about 200 K at 800–1000 km depth. Duba et al.'s (1976) selenotherm (Fig. 10) results in excessive strain-rates for all lithospheric models (Fig. 12). If Post's (1977) parameters result in strain-rates that are too high, then the temperatures must be reduced even further.

(iii) Flow laws cannot be scaled to the lunar interior from the laboratory conditions of high stress and strain-rates. The validity of this extrapolation involves the question of whether the dominant deformation mechanisms in the laboratory experiments still adequately describe the deformations on the geological time scale (see Paterson, 1976). Possibly the power-law creep mechanism may be less important than Coble creep at the relatively low stresses and temperatures in the Moon (see, for example, the “deformation mechanism” maps of Stocker and Ashby (1973) and Carter (1976). But in this case the dislocation creep laws will underestimate the flow rates, thereby exacerbating the temperature problem.

In view of the interdependence of these arguments it is not possible to draw firm conclusions about the relevant parameters describing the lunar interior. What does seem certain is that the lunar temperatures must be lower than suggested by the recent interpretations of the heat-flow and conductivity measurements for, if not, the stresses associated with the lunar mass concentrations would have relaxed by now. If we assume that the seismically defined high-Q and high S-velocity layer (Model M2A) also corresponds to the layer capable of supporting some stress for long periods, then the temperatures throughout much of the upper mantle cannot exceed \(0.7 T_m\), even for the low strain-rate flows predicted by Post's (1977) results. The proposed temperature profile is quite similar to that estimated by Toksöz et al. (1978) from thermal evolution calculations.

In concluding that the temperatures in the lunar interior are relatively low, or that viscosities are high
Fig. 12. Left: strain rates for the nominal temperature profile, Post's (1977) flow law, and stresses associated with the gravity field for the various lunar models discussed in the text. Centre: strain rates for three different selenotherms: (i) nominal profile, (ii) Duba et al. (1976), (iii) temperatures about 100 K lower than the nominal profile. Right: strain rates for three different models and Duba et al.'s (1976) selenotherm.

(>10^{26} or 10^{27} poise) it is perhaps useful to recall some of the assumptions made and the limitations of the data sets:

(a) the adopted gravity spectrum, while containing considerably less power than many of the earlier models, may still overestimate the power. If this is so, the stress-differences will be somewhat underestimated.

(b) In inverting the gravity for density anomalies and stress-state, it was concluded that much of the gravity originated in the crust and upper mantle. However, lower regions may not be entirely devoid of lateral density variations, but they appear to be sufficiently small not to contribute much of the potential spectrum because of the upwards attenuation effect. Hence stress-differences may be underestimated.

(c) The stresses have been computed separately for each harmonic and summed according to either eq. 9a or 9b. The preceding discussion was based on eq. 9a and at any point the actual shear stress may exceed this limit. Expression 9b provides an upper limit and, if this is used, the lunar interior temperatures must be even further reduced.

(d) The nature of the potential spectrum suggests that only part of the power in the \( l = 2 \) harmonics originates in the same near surface anomalous density layer as does the rest of the field, and that the power above this “decay” spectrum (Fig. 3) may have a distinctly different cause. The above stress calculations contain all of the power in the second-degree harmonics. The main effect of these terms is not so much to increase the maximum stresses themselves but to increase the depth down to which the maximum stresses occur (Fig. 9). If the “anomalous” part of this bulge is to be attributed to density anomalies
associated with a second degree convection pattern in the deep interior (Runcorn, 1974, 1977) the strain-rates as functions of depth (Fig. 11) will be reduced by about one order of magnitude for the models M2 and M3 and less for the thinner lithosphere models such as M2A.

(e) The estimates of the strain-rates occurring in the Moon are based on the rather ad-hoc interpretation of the lunar topography spectrum. The lack of tectonism on the Moon supports the contention that these strain rates are much lower than on Earth. If the relaxation occurred rapidly during some early phase of the Moon's history the present strain rates would be even lower and the temperatures would be further reduced.

(f) A central assumption is that the density anomalies and associated stresses have persisted for at least $3 \cdot 10^7$ y following the end of the period of mare volcanism. This is suggested by the absence of subsequent magmatic and tectonic activity.

(g) The extrapolation of flow-law parameters from the laboratory to the Moon remains one the biggest sources of uncertainty. The above deductions about the temparature are based on the parameters by Post (1977) that give the lowest strain-rates and any one of the other set of parameters summarized in Table II requires further reductions in temperature.

References


