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THE LUNAR FOSSIL BULGE HYPOTHESIS REVISITED

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The lunar gravity field can be satisfactorily explained by relatively-small wavelength, density anomalies located near the lunar surface. The exceptions to this are the second degree harmonics and we postulate a dual origin for these terms; they are partly a consequence of the same near-surface anomalies as the remainder of the gravity field and partly a measure of a fossil bulge. This latter contribution represents an equilibrium condition preserved from a time when the synchronously rotating Moon was approximately 25 Earth-radii from the Earth. If this shape was acquired at some time soon after $4.0 \cdot 10^9$ years ago, then the rate of dissipation of tidal energy in the Earth has been much lower in the past then it is now. The implications of this on the evolution of the Earth–Moon system are briefly discussed.

1. Introduction

A hypothesis sometimes discussed in the lunar science literature, and which apparently was first investigated in detail by Jeffreys in 1915, is that the present shape and mass distribution of the Moon represent an equilibrium state at a time when it was closer to the Earth than it is now; when both the Earth's tidal force and the lunar spin were greater than present values. These studies have generally indicated that the present shape corresponds to an equilibrium form for a synchronously rotating Moon at 20–30 Earth-radii from the Earth (Jeffreys, 1915, 1937; Kopal, 1969). These studies were based on the moments of inertia of the Moon, derived from the libration constants, and suffered from an inadequate data set as well as from the fact that the solutions were not self-consistent. A similar calculation can be made using the second-degree lunar potential coefficients, provided that it can be demonstrated that they are sufficiently different from the other higher degree coefficients so as to merit a special interpretation.

Two considerations lead us to at least consider the possibility that there is something unusual about the second-degree harmonics of the lunar gravity field. Firstly, the visual inspection (Fig. 1) fo the non-dimensional spectrum of the lunar gravitational potential

$$V_l^2(\Delta U) = (\bar{C}_{lm}^2 + \bar{S}_{lm}^2)\,, \tag{1}$$

where $\bar{C}_{lm}$ and $\bar{S}_{lm}$ are the fully normalized potential coefficients, indicates that the power at degree $l = 2$ exceeds that for higher l by a magnitude or more. Secondly, the decay of the spectrum is generally compatible with density anomalies in the upper part of the lunar mantle or crust except for $l = 2$, and no distribution of small-wavelength density anomalies of a random magnitude distribution can explain the spectrum (Fig. 1). We therefore argue that part of the second degree harmonics in the potential has the same origin as the higher degree harmonics of the field while the difference between the observed power and this "predicted" power may be associated with a fossil bulge.

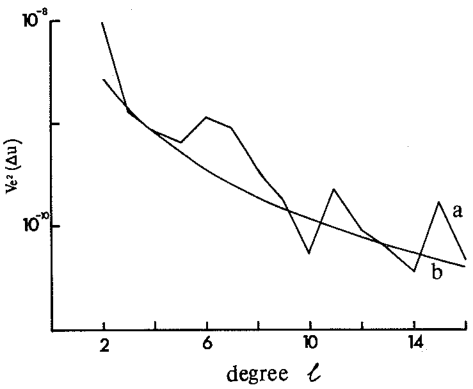

Fig. 1. The average of the power spectra of the lunar gravitational potential according to Ferrari (1977) and Ananda (1977) (a), and the spectrum resulting from a near-surface distribution of small wavelength density anomalies (b).

2. The anomalous bulge

The gravitational spectrum illustrated in Fig. 1, is based on a smoothed average of two recent solutions of the lunar gravity by Ananda (1977) and Ferrari (1977) and is discussed further by Lambeck and Pullan (1980). For the Earth the comparable spectrum decays approximately by:

$$V_l^2(\Delta U) = (2l+1)\,10^{-10}\,l^{-4}$$

and this can be attributed to density anomalies distributed randomly in depth as well as laterally and with dimensions small compared with $2\pi R/l$, R being the planetary radius (Lambeck, 1976). The lunar spectrum decays more slowly than this rule and is indicative of a near-surface origin for the density anomalies. The exception to this is the $l = 2$ spectral estimate, and no simple distribution of randomly distributed density anomalies can explain the entire spectrum. If the random density model is fitted to the spectral estimates for $l \geqslant 3$ and extrapolated back to $l = 2$, we obtain a "predicted" estimate of the power in the $l = 2$ harmonics if these harmonics had the same origin as the $l \geqslant 3$ harmonics. The difference between the observed $V_2^2(\Delta U)$ (defined by eq. 1, and referred to the present hydrostatic equilibrium figure) and the predicted value is:

$$V_2^2(\epsilon_U) = [V_2^2(\Delta U)]_{\text{observed}} - [V_2^2(\Delta U)]_{\text{predicted}}$$

$$\sim 6.8 \cdot 10^{-9} \qquad (2)$$

and it is this part that is "abnormal" and merits a special interpretation.

The equilibrium form of the Moon, rotating synchronously such that at all times its mean motion n equals its spin velocity θ about its center of mass, and acted upon by the Earth's gravitational attraction, is given in terms of the potential coefficients (e.g. Kopal, 1969) as:

$$\sqrt{5}\cdot \bar{C}_{20}^{HE} = -\frac{C-(1/2)(A+B)}{mR^2} = -\frac{5}{6}\left(\frac{R}{a}\right)^3 \frac{M}{m}(\Delta - 1) \qquad (3a)$$

$$\sqrt{\tfrac{5}{12}}\cdot \bar{C}_{20}^{HE} = -\frac{A-B}{4mR^2} = \frac{1}{4}\left(\frac{R}{a}\right)^3 \frac{M}{m}(\Delta - 1) \qquad (3b)$$

where A, B, C are the principal moments of inertia with $C > B > A$, M is the mass of the Earth, m the mass of the Moon, a is the mean Earth–Moon distance, and Δ is a factor to correct for any departure from the homogeneous density distribution. The $\bar{C}_{20}$ and $\bar{C}_{22}$ are fully normalized. For realistic lunar models in which the Radau approximation is valid, $\Delta \sim 2.48$, compared with 2.50 for a homogeneous model. From the hydrostatic theory (eq. 3) we may derive:

$$V_2^2(\Delta U^{HE}) = \frac{13}{45}\left(\frac{R}{a}\right)^6\left(\frac{M}{m}\right)^2 (\Delta - 1)^2$$

$$\sim 4.2 \cdot 10^3 \left(\frac{R}{a}\right)^6 \qquad (4)$$

Equating eq. 2 with eq. 4 gives:

$$a \sim 25\, R_{\mathrm{E}} \qquad (5)$$

where R_{E} denotes the radius of the Earth.

Jeffreys (1915) obtained 23 R_{E} by comparing the then available libration constant β with the hydrostatic theory. But he was unsatisfied with this result because the observed value for f defined by:

$$f = (C-B)/(C-A) = (\bar{C}_{20} + \bar{C}_{22}/\sqrt{3})/(\bar{C}_{20} - \bar{C}_{22}/\sqrt{3}) \qquad (6)$$

which is independent of the Earth–Moon distance, was not in agreement with that predicted by the hydrostatic theory. What we argue here is that part of the observed $\bar{C}_{20}$ and $\bar{C}_{22}$ have the same origin as the remainder of the gravity field perturbations, and that only the difference ϵ_U may have a special significance – that it reflects the equilibrium shape of the Moon at some time in the past.

Goldreich and Toomre (1969) discussed a similar fossil bulge problem for the Earth, namely whether or not the non-hydrostatic part of the $\bar{C}_{20}$ term is significantly larger than other terms in the potential expansion. They computed the ratio f, defined by eq. 6, for some 2000 randomly generated models of a nearly spherical body upon which surface mass anomalies were superimposed in a random manner. They concluded that, for the Earth, the observed value of $f \sim 0.47$ is what can be expected for such models. They also noted that for the Moon, $f \sim 0.65$, again not unpredictable. Thus, for the Moon, like the Earth, the non-hydrostatic part of $\bar{C}_{20}$ is not exceptionally large when compared with $\bar{C}_{22}$. However, whereas for the Earth there is no a priori reason why $\bar{C}_{22}$ should be relatively large, there is a good reason why it should be so for the Moon in view of the hydrostatic equilibrium conditions given by eq. 3. For the Moon the question is, are both $\bar{C}_{22}$ and $\bar{C}_{20}$ sufficiently large (relative to the other potential coefficients) to merit a special interpretation? We argue that it is. The Goldreich and Toomre (1969) argument is not applicable in this case.

Runcorn (1974, 1977) objects to the fossil bulge hypothesis and prefers to explain the large second-degree coefficients in the lunar potential as a consequence of thermal convection. The seismically determined thick lithosphere and the presence of deep focus moonquakes implies that if convection occurs it will only occur at depths below ~800 km or more. However, Runcorn assumes that only the outer 200 km is capable of supporting stress-differences of the order of 50 bar for long periods and that convection must therefore be mantlewide. Why the convection pattern should exhibit a stable second-degree harmonic configuration is obscure, but possibly this condition is not essential since the contribution of any higher harmonics to surface gravity is much reduced due to the attenuation of the potential with distance from the anomalous density source. Runcorn's objections to the fossil bulge are threefold. One is the discrepancy between the observed and hydrostatic values for the ratio f defined by eq. 6. We do not consider this a serious objection since we attribute only part of the second-degree harmonics to the fossil bulge. Another objection is that the geometrical and dynamical shapes of the Moon do not bear the relation expected from the fossil bulge model. Again we do not consider this important since we consider the second-degree harmonics in both the topography and gravity to be a consequence of two different contributions. The reduced potential (eq. 2) implies a power of the topography $V_l^2(h)$ (defined similarly to eq. 1) is:

$$[V_2^2(h)]_{\text{reduced}} = \left[\frac{5}{3}\cdot\left(\frac{\bar{\rho}}{\rho_c}\right)\right]^2 [V_2^2(\Delta U)]_{\text{reduced}}$$

$$\sim 2.3 \cdot 10^{-8}$$

compared with an observed power which is three times larger than this. $\bar{\rho}$ is the mean density of the planet and ρ_c the density of the topography. The difference

$$[V_2^2(h)]_{\text{observed}} - [V_2^2(h)]_{\text{reduced}} = 4.6 \cdot 10^{-8}$$

is of the same magnitude as the power in harmonics $l = 3$ or 4 and higher and will be the consequence of the same random events forming the lunar topography described by these higher degrees. A third objection by Runcorn is that lunar materials will creep when subjected for long time periods to the stresses associated with the fossil bulge. This is a more serious objection for, in equating the observed excess bulge with some past equilibrium shape, we have implied that the Moon has sufficiently-high finite strength for it to support nonhydrostatic stresses over $3 \cdot 10^9$ y or longer (see below).

Recently, Anderson (1978) has developed the fossil bulge argument further by comparing the ellipticities of surfaces fitted separately to the lunar maria and highlands. He concludes that the latter are smaller and argues that the Moon was captured near the time of the mare volcanism, when it was closer to the Earth than at the time of the highland formation. The limited topographic data available does not permit such conclusions to be drawn, but in any case we would argue that this result is probably fortuitous in that part of the equipotential bulge will be tidal in origin and part will be a consequence of the small-wavelength anomalies. This latter part exceeds the difference in ellipticities found by Anderson (1978).

3. Stress considerations

The stress-differences required to support the near-surface density anomalies implied by the gravity field have been computed by Lambeck and Pullan (1980) and average values do not appear to exceed more than a few tens of bars. These calculations have been carried out for several lunar models and Fig. 2 illustrates results for two of them. Model M2 comprises a 1000 km thick lithosphere overlying a considerably weaker lower mantle. The outer layer includes the very high Q zone (Nakamura et al. 1976; Goins et al. 1978) and the region of somewhat greater, but still high attenuation in which the S-wave velocity gradient is negative. It extends down to the greatest depth at which moonquakes have been recorded. The lithosphere in model M2A is only 500 km thick and a 100 km transition layer separates it from the much weaker lower region. The stress-differences τ_{lm} in the planet for the individual harmonics of degree l and order m have been summed according to the expression:

$$\tau(r) = \{\sum_l \sum_m \tau_{lm}^2(r)\}^{1/2}$$

rather than by computing the stresses for each harmonic as a function of latitude and longitude, and only then summing over degree and order to find the maximum differences. Thus $\tau(r)$ is a measure of the average stress-difference rather than of a maximum value. The stress calculation summarized in Fig. 2, includes the contribution from the anomalous part of the bulge and from the higher-degree harmonics.

If we consider only that part of the bulge that has the same anomalous density source as the remainder of the field, then the maximum values of the computed stress-differences are not changed significantly but, for models with thick lithospheres such as M2, the stress-differences deeper in the planet are decreased. Whether or not these stresses can be supported by the finite strength of the lunar materials depends on the temperature and creep rates in the lunar interior. Neither the selenotherm nor the appropriate flow mechanism can be observed directly, but we have previously argued (Lambeck and Pullan, 1980) that the stress-differences associated with the higher-wavelength harmonics in gravity imply lower temperatures than suggested by Duba et al. (1976)

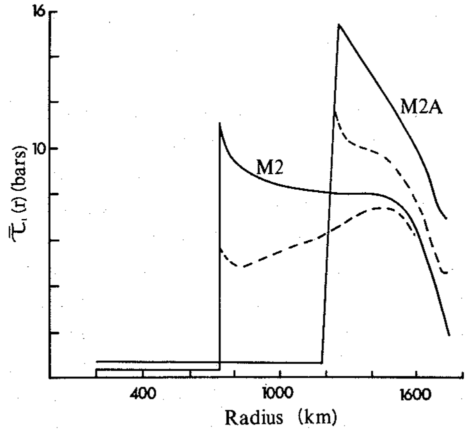

Fig. 2. Stress-differences in the Moon, according to the models M2 and M2A, associated with the lunar gravity anomalies. The dashed lines represent the results if the anomalous part of the bulge is excluded from the gravity anomalies.

and at most near the lower limit proposed by Keihm and Langseth (1977) at 300 km depth. Hence creep rates may be quite low, in particular if Post's (1977) experimental results for dry dunite are appropriate to the lunar interior. If not, it is difficult to reconcile

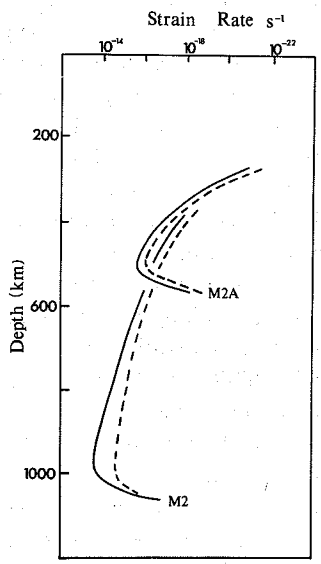

Fig. 3. Strain rates for the temperature profile of Lambeck and Pullan (1980) and stresses associated with the gravity field, for the two lunar models discussed in the text. The dashed lines are for the strain rates when the anomalous bulge is excluded.

this with the observation that the shorter wavelength anomalies associated with features such as the circular maria, have persisted for as long as they have, without invoking some sort of dynamic support.

Once stress-differences, flow laws and a selenotherm have been adopted, strain rates can be computed and compared with the maximum observed strain-rates. Figure 3 gives results for models M2 and M2A, the rheology of Post (1977) and the selenotherm proposed by Lambeck and Pullan (1980). The exclusion of the "fossil" part of the bulge from these calculations reduces the strain-rates by at most one order of magnitude (Fig. 3) and this is less than some of the other uncertainties introduced by the flow laws and selenotherm. It does not appear warranted to make any distinction between the nature of support of the two contributions to the bulge; of the density anomalies associated with harmonics of degree greater than two, if those that contribute to the mascons are supported by a finite strength, then the excess bulge can also be supported in the same way.

4. Implications for the evolution of the Earth–Moon system

The last major tectonic activity on the Moon occurred about $3.2 \cdot 10^9$ y BP (before present) with the conclusion of a period of extensive eruption of mare basalts. Prior to this the lunar surface had been subjected to intense bombardment which reached a climax at about $3.9 \cdot 10^9$ y BP (e.g. Wasserburg et al. 1977). The last three aeons have been times of relative quiescence. The mare formation is probably of lesser significance in so far as contributing to the global shape of bulge since (a) the mare basalts comprise less than one percent of the total mass of Moon, and (b) their formation were local rather than global events and occurred over a time span of nearly 10^9 years. However, the flooding may be indicative of the time when the Moon acquired its final shape. The basalt flooding was presumably triggered by the large impacts at about $4.0 \cdot 10^9$ y BP and the activity was greatest immediately after this time. Subsequent flooding became increasingly less important as the zones of weakness, generated by the impacts, gradually healed and the upper mantle cooled by conduction. By about $3.2 \cdot 10^9$ y BP the crust and outer mantle formed an effective barrier to further intensive volcanism. Thus from the results of eq. 5, one may draw the tentative conclusion that the Moon was at 25 R_E from the Earth at some time between ~4.0–$3.2 \cdot 10^9$ y BP, with a distinct preference for the earlier date since the large gravity anomalies over the circular mare are indicative that the lunar mantle could support significant non-hydrostatic stresses soon after the end of the bombardment.

The rate of change of the semi-major axis of the lunar orbit for a frequency independent Q is given approximately (e.g. Jeffreys, 1970; Lambeck, 1977) by the expression:

$$\dot{a} = A(k_2/Q)a^{-11/2} \tag{7}$$

where:

$$A \sim 3Gm\,R^5/[G(M+m)]^{1/2}$$

and k_2 and Q are the tidal effective Love number and specific dissipation function of the Earth. If ϵ is the lag angle between the tidal potential and the Earth's delayed response, Q^{-1} — $\sin \epsilon$. Integrating eq. 7 from the present T, where $a \sim 60\,R_E$, to some time (T) in the past gives:

$$[(a)_T]^{13/2} = [(a)_{T_0}]^{13/2} + (13/2)(k_2/Q)A(T - T_0)\,.$$

For $[(a)_T] \sim 25\,R_E$ at $T_0 = -3.9 \cdot 10^9$ years, the average measure of the tidal response is:

$$k_2/Q \sim 0.010$$

or, for $k_2 \sim 0.30$, $Q^{-1} \sim 0.03$, and $\epsilon \sim 1.9°$. If we assume that the Moon acquired its present shape some $3.2 \cdot 10^9$ years ago, $\epsilon \sim 2.3°$.

Because tidal evolution of the semi-major axis is inversely proportional to $(a)^{11/2}$, the time required for the orbit to evolve from a very close-Earth approach to 25 R_E is short (less than 0.1 aeons). Thus even with the constraint provided by the fossil bulge hypothesis, a close encounter of the Moon with the Earth some 3–$4 \cdot 10^9$ y BP ago cannot be avoided unless the equivalent lag angles have been variable with time, and the value some 4 aeons ago was less than the average for the subsequent time interval. The impact of bodies larger than those that formed Imbrium or Orientale, prior to $4.0 \cdot 10^9$ y BP ago, presumably had a much more important effect on the early evolution of the lunar orbit than did tidal dissipation.

The equivalent phase lags for the past 3000 years, based on ancient eclipse records and on modern telescope observations, is about 5–6° (Lambeck, 1977; 1980), while the average value over the last $4 \cdot 10^8$ y, based on the fossil coral and bivalve record, appears to have been about 4° (Lambeck, 1978; 1980). As most of the dissipation of tidal energy now occurs in the oceans, the variation in the lag angles can be readily attributed to changes in the ocean configuration and to changes in ocean volume. A comparison of the three estimates of lag angles averaged over different time intervals suggests in fact an increase in the dissipation with time. A number of factors may contribute to this. The free oscillation periods of the ocean are given very approximately by $2L/\sqrt{gD}$ where L is the typical horizontal length scale of the ocean and D the depth. For $L \sim 4000$ km and $D \sim 3$ km, the resonance period would be about 13 hours – close to the period of the forcing function of the present principal semi-diurnal lunar tide. If sea level was lowered, or if the continents were grouped together, the resonance period would lie further away from the forcing function period. In both cases, dissipation would be reduced. Thus periods of rapid seafloor spreading such that the displaced waters invaded the continents, as occurred globally during the Cretaceous, may have coincided with periods of reduced dissipation, even though the extent of shallow seas was greater than it is now. That extensive shallow seas do not necessarily imply high dissipation rates appears to be supported by the tidal calculations of Sünderman and Brosche (1978) for the Permian. If the loss of tidal energy occurs by friction on the sea floor, shallow seas alone are not enough to increase dissipation; deep ocean tides are also required in order to force the water over the shallow sea floor.

Periods during which the continents were distributed with less longitudinal variability than now, as may have occurred during the Cambrian and Ordovician and again later, when the Tethys Sea formed a large equatorial ocean, may have been associated with less dissipation than at present. Likewise, a grouping of continents along an equatorial belt during the Late Proterozoic (e.g. Piper, 1978) can also reduce dissipation.

These general arguments suffice to illustrate that some temporal variability in the equivalent lag angles will most likely have occurred during the last 3 aeons and that the present rate of dissipation may be quite atypical. If, at the same time, there has been a gradual evolution of the extent of the ocean basins, it is quite possible that the value for the lag angle some three billion years ago was actually less than the average value. At present it is not very difficult to accommodate a situation in which close approach never occurred at all.

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