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Deglaciation effects on the rotation of the Earth

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Summary. The viscoelastic response of the Earth to the mass displacements caused by late Pleistocene deglaciation and concomitant sea level changes is shown to be capable of producing the secular motion of the Earth's rotation pole as deduced from astronomical observations. The calculations for a viscoelastic Earth yield a secular motion in the direction of 72° W meridian which is in excellent agreement with observed values. The average Newtonian viscosity and the relaxation time obtained from polar motion data are about $(1.1 \pm 0.6)\,10^{23}$ poise (P) and $10^4(1 \pm 0.5)$ yr. The non-tidal secular acceleration of the Earth can also be attributed to the viscoelastic response to deglaciation and results in an independent viscosity estimate of 1.6×10^{23} P with upper and lower limits of 1.1×10^{23} and 2.8×10^{23} P. These values are in agreement with those based on the polar drift analysis and indicate an average mantle viscosity of $1–2 \times 10^{23}$ P.

1 Introduction

Analyses of the International Latitude Service data indicates that the Earth's mean pole exhibits a secular drift with a magnitude of $0.003'' \sim 0.004''\,\mathrm{yr}^{-1}$ in the direction of $65° \sim 73°$ W (Lambeck 1980). The possibility that this secular drift is merely an apparent displacement of the pole due to motions of ILS stations located on different tectonic plates has been investigated and dismissed by Dickman (1977) and Soler & Mueller (1978) and the plate motions can account for only about 10 to 20 per cent of the observed value.

A plausible geophysical mechanism for the polar drift would be one that introduced changes in the products of inertia of the Earth due to large scale mass displacements, or one which involves small masses moving about in an aperiodic manner so as to generate appreciable changes in the angular momentum of the Earth. The former, involving mass displacements of large horizontal extent seem to be the more plausible trigger for the polar drift. While mass displacements associated with tectonic plate movements and mantle convection could introduce a time variant inertia tensor, there is no way in which this can be

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evaluated while the convection problem remains unsolved. The more tractable effects of deglaciation and associated sea level changes have been investigated by Munk & MacDonald (1960) and Lambeck (1980) for an elastic Earth assuming a uniform rise in sea level over the oceans. While these studies have indicated that the direction of the drift is roughly compatible with this mechanism, the magnitude of the drift cannot be established from the available data on ice volumes.

In the present work deglaciation of the Laurentide and Fennoscandinavian ice sheets and the Earth's viscoelastic response to the concomitant ice and water surface mass redistribution is examined as a possible cause of the polar drift. The analysis is extended to an estimation of the Earth's mean viscosity as well as to the investigation of the non-tidal secular acceleration of the Earth's rotation due to deglaciation. Recent analysis of ancient eclipse data (see Lambeck 1980, for a review) as well as the inclusion of the gravitational effect of the inviscid core on the effective relaxation time lead to somewhat different results than obtained previously by O'Connell (1971).

2 Sea level changes due to deglaciation

The earlier works pertaining to the effects of deglaciation on the rotation of the Earth were based on the assumption that changes in sea level are uniform over the oceans. However, Farrell & Clark (1976) showed that this assumption results in errors by as much as 25 ~ 50 per cent over large areas of the oceans. Such discrepancies between the actual sea level rises and the eustatic change* stem from the fact that the static ocean surface must be an equipotential and remain so after the sea level rise. Also, the decrease in the mass and the attraction of the melting ice actually causes the sea level to drop in the vicinity of the ice sheet.

To compute the effect of the sea level change on the Earth's rotation, the ocean–ice redistribution is first considered on a rigid Earth and the consequence of this on the rotation is then evaluated by using the appropriate Love numbers. The sea level change on a rigid Earth is readily computed from the first-order solution of the integral equation given by Farrell & Clark (1976) who also developed the governing integral equation for an elastic Earth. The relevant equation is

$$h(\theta,\lambda,t)=\frac{\rho_i}{g}\int_{\Omega_i}F(\psi)I\,d\Omega-\frac{M_i}{\Omega_o\rho_\omega}-\frac{\rho_i}{g}\left\langle\int_{\Omega_i}F(\psi)I\,d\Omega\right\rangle$$
$$-\frac{\rho_\omega}{g}\left\langle\int_{\Omega_o}F(\psi)h\,d\Omega\right\rangle+\frac{\rho_\omega}{g}\int_{\Omega_o}F(\psi)h\,d\Omega, \tag{1}$$

where: $h(\theta, \lambda, t)$ is the static change in sea level at time t of an ocean point of colatitude θ, longitude λ; ρ_i, ρ_ω are the densities of ice and water; $F(\psi) = Rg/2M_e \sin\psi/2$, is the appropriate Green's function for the rigid Earth; $\psi = \cos^{-1}[\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\lambda-\lambda')]$ is the geocentric angle between the fixed and moving points; R, g and M_e are the mean radius, gravity and mass of the Earth, I is the change in the ice thickness, taken negative for melting; M_i is the total mass of melting ice, again taken as negative for melting, Ω_i, Ω_o are the surface areas of the ice sheets and the oceans; $d\Omega = R^2\sin\theta'\,d\theta'\,d\lambda'$; $\langle\,\rangle$ indicates the mean value of a quantity over the oceans.

The solution of the integral equation (1) can be obtained by integration in which the eustatic rise of sea level is taken as the initial approximation for h. Farrell & Clark (1976)

* The term eustatic sea level is used here to mean $\Delta V/A$ where ΔV is the change in volume of the ocean and A the area of the ocean surface.

have shown that this iteration converges rapidly and the results of the first iteration do not differ from the final solution by more than 10 per cent. The first-order solution is

$$h(\theta,\lambda,t)=\frac{\rho_i}{4\pi\rho_\omega a_0^0}\int_{\Omega_i}\zeta\,d\sigma+\frac{3\rho_i}{8\pi\rho_e}\left[-\left(\int_{\Omega_i}\frac{\zeta}{\sin\psi/2}d\sigma-\left\langle\int_{\Omega_i}\frac{\zeta}{\sin\psi/2}d\sigma\right\rangle\right)\right.$$
$$\left.+\left(\frac{1}{4\pi a_0^0}\int_{\Omega_i}\zeta\,d\sigma\right)\left(\int_{\Omega_o}\frac{d\sigma}{\sin\psi/2}-\left\langle\int_{\Omega_o}\frac{d\sigma}{\sin\psi/2}\right\rangle\right)\right], \tag{2}$$

where: ρ_e is the mean density of the Earth; a_0^0 is the zero degree harmonic coefficient of the ocean function; ζ is the change in the height of the ice sheet due to melting at point (θ',λ') at the instant t, i.e. $\zeta=-I$. The first term on the right-hand side is the eustatic rise in sea level. The time unit is taken to be 1 yr and the dynamic changes in sea level at shorter time periods are not relevant to this study. The conservation of mass is automatically satisfied in this solution.

The right-hand side of equation (2) can be computed by numerical integration once the ice-melting history during deglaciation is specified. Peltier & Andrews (1976) have compiled the spatial and temporal distribution of the Laurentide and Fennoscandinavian ice sheets which were the major ice masses that disintegrated with time. The ice thickness averaged over $5^\circ\times5^\circ$ areas are given by Peltier & Andrews at four epochs from which the average rate of change of ice thickness over $10^\circ\times10^\circ$ areas has been obtained. In view of other uncertainties in the data a simple model based on a linear variation of height with time appears to be sufficiently accurate and a contoured form of these average rates at 10° grids is illustrated in Fig. 1.

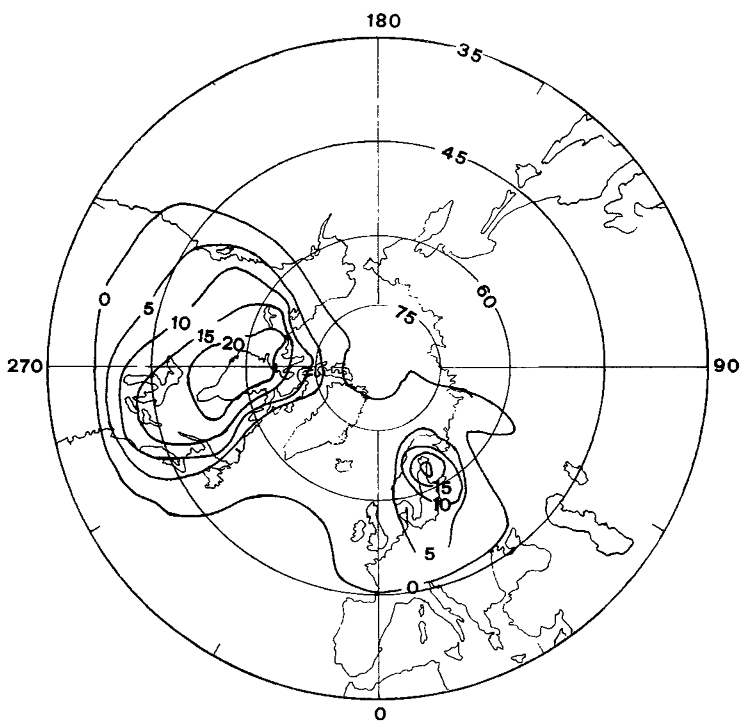

Figure 1. Average rate of change of glacier heights from 18 000 to 6000 yr ago.

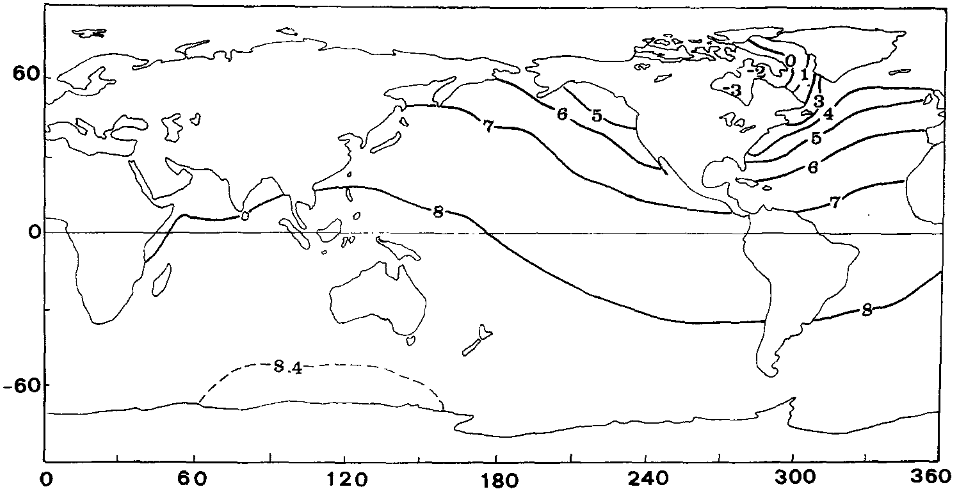

Figure 2. Average annual changes in sea level during the deglaciation period on the assumption that the Earth is rigid. Contour interval is 1 mm yr^{-1}. An average rate of 1 mm yr^{-1} corresponds to a total change of 12 m.

With this ice history, the ocean function of Balmino, Lambeck & Kaula (1973) and equation (2), the average yearly changes of sea level have been computed for $10^\circ \times 10^\circ$ mean ocean areas between the start of the final disintegration of the ice sheets at epoch 18KYBP (kyr before the present) to the end of melting at 5KYBP. Fig. 2 summarizes the results for rigid Earth as the average annual change of sea level during the 12 000 yr of deglaciation. Fig. 2 indicates that the rise in sea level is far from uniform, the total varying from 96 m in the South Pacific to about 50 m in the Atlantic, west of Europe.

3 The excitation function for a rigid Earth

The quantitative estimates of surface mass displacements involved in deglaciation and associated sea level changes were obtained in the previous section on the assumption that the solid Earth does not deform under the changing load. The consequences of instantaneous and long term deformation of the solid Earth under the surface load will be considered in the next section as modifications of the results of this and the preceeding section.

The general perturbation equations of rotational motion of a body are given by Munk & MacDonald (1960) as

$$\frac{\dot{m}_1}{\sigma_r} + m_2 = \psi_2$$

$$\frac{\dot{m}_2}{\sigma_r} - m_1 = \psi_1 \qquad (3)$$

$$\dot{m}_3 = \psi_3$$

where: $m_i (i = 1, 2, 3)$ are the direction cosines of the instantaneous rotation axis; σ_r is the Eulerian frequency of free wobble; $\psi_i (i = 1, 2, 3)$ are the so-called excitation functions incorporating in general the effects of mass displacements, relative motion, or applied torques; and ($\dot{}$) denotes the time derivative of a quantity.

The excitation functions due to a gradual redistribution of the surface masses in rigid Earth are given as (Munk & MacDonald 1960)

$$\psi_1 = \frac{c_{13}}{C-A} + \frac{\dot{c}_{23}}{\omega(C-A)},$$

$$\psi_2 = \frac{c_{23}}{C-A} - \frac{\dot{c}_{13}}{\omega(C-A)}, \tag{4}$$

$$\psi_3 = -\frac{c_{33}}{C},$$

where: C and A are the maximum and minimum moments of inertia of the Earth; ω is the rotation rate of the Earth; c_{ij} are the perturbations in the inertia tensor for a surface load $q(t)$, or

$$-c_{ij}(t) = \int_{\Omega} q(t)\,[\delta_{ij}x_k(t)x_k(t) - x_i(t)x_j(t)]\,d\Omega \qquad (i, j = 1, 2, 3); \tag{5}$$

Ω is the surface area of the Earth.

Taking the start of glacial melting as the time of origin and denoting the end of the deglaciation period by t_0, the excitation function pertaining to the present problem can be written as

$$\psi_i = {}_0\dot{\psi}_i t_0 \left[1 - H(t_0 - t) + \frac{t}{t_0} H(t_0 - t)\right], \tag{6}$$

where

$$H(t_0 - t) = \begin{cases} 1 & 0 < t < t_0 \\ 0 & t > t_0 \\ \tfrac{1}{2} & t = t_0, \end{cases}$$

is the Heaviside unit step function.

The rate of change of the inertia elements can be computed from equation (5) by numerical integration of the ice heights and sea level changes. The results in units of yr^{-1}, are

$$\dot{c}_{13}/(C-A) = 3.80 \times 10^{-8}, \quad \dot{c}_{23}/(C-A) = -11.5 \times 10^{-8}, \quad \dot{c}_{33}/C = 0.058 \times 10^{-8}. \tag{7}$$

4 Polar drift on viscoelastic Earth

The mechanical response of the Earth depends, besides other factors, on the time-scale involved. Although the Earth responds elastically to excitations of high frequency (the seismic and tide range), its behaviour in the intermediate frequency range ($\sim 10^{-4}\,\text{yr}^{-1}$) is often considered to be that of a viscoelastic Maxwell body. Laboratory work on mantle-like materials indicate that at high temperatures, strain-rates are proportional to some power of the deviatoric stresses (see Carter 1976, for a recent review of experimental results), however, we assume a linear stress–strain rheology in that this appears to adequately describe the rebound response for which the deviatoric stresses are small compared with convective stresses (Cathles 1975; Peltier, Farrell & Clark 1978; Kaula 1979; but see also Post & Griggs 1973; Brennen 1974) and also because this choice renders the problem readily tractible.

The equations of rotational motion which are relevant to polar motion on the viscoelastic Earth follow from equation (3) following the procedure developed by Munk & MacDonald (1960), namely

$$\dot{\mathrm{m}} - i\sigma_{\mathrm{r}}(1 - \hat{k}/k_{\mathrm{f}})\mathrm{m} = -i\sigma_{\mathrm{r}}(1 + \hat{k}')\psi_{\mathrm{r}}, \tag{8}$$

where: $\mathrm{m} = m_1 + im_2$; $\psi = \psi_1 + i\psi_2$ is the complex excitation function for the rigid Earth; k_{f} is the fluid Love number; $\hat{k}, \hat{k}'$ are the Love operators. For a Maxwell visco-elastic solid

$$\hat{k} = \frac{k_{\mathrm{f}}}{1 + \hat{\mu}} = \frac{k_{\mathrm{f}}}{1 + \hat{\mu}}\left(1 + \frac{\mu\alpha}{\hat{D} + \alpha}\right)$$
$$\hat{k}' = \frac{1 - l_{\mathrm{s}}}{1 + \hat{\mu}} = -\frac{1 - l_{\mathrm{s}}}{1 + \hat{\mu}}\left(1 + \frac{\mu\alpha}{\hat{D} + \alpha}\right) \tag{9}$$

Here $\hat{\mu}$ and l_{s} are the dimensionless rigidity operator and the isostatic factor respectively. For an effective average rigidity μ', or a dimensionless rigidity $\mu = (19/2)\mu'/\rho g R$

$$\hat{\mu} = \frac{\mu D}{D + \tau^{-1}}, \tag{10}$$

where τ is the relaxation time which is simply the viscosity η divided by rigidity, $D = d/dt$ is the differential operator, and

$$\alpha^{-1} = (1 + \mu)\tau$$
$$\gamma = l_{\mathrm{s}}/(l_{\mathrm{s}} + \mu)\tau. \tag{11}$$

Substituting equations (9) and (11) into equation (3) yields

$$\dot{\mathrm{m}} - i\sigma_{\mathrm{r}}\frac{\mu}{1 + \mu}\frac{D}{D + \alpha}\mathrm{m} = -i\sigma_{\mathrm{r}}\frac{l_{\mathrm{s}} + \mu}{1 + \mu}\frac{D + \gamma}{D + \alpha}\psi.$$

This equation can be simplified by observing that $(l_{\mathrm{s}} + \mu)/(1 + \mu) \simeq \mu/(1 + \mu)$, and the frequency of the Chandler wobble is $\sigma_0 = \sigma_{\mathrm{r}}(1 - k_2/k_{\mathrm{f}}) = \sigma_{\mathrm{r}}\mu/(1 + \mu)$. Hence

$$\frac{D(D + \alpha - i\sigma_0)}{D + \alpha}\mathrm{m} = -i\sigma_0\frac{D + \gamma}{D + \alpha}\psi.$$

whose solution is

$$\mathrm{m} = -i\sigma_0\frac{D + \gamma}{D(D + \alpha - i\sigma_0)}\psi,$$

or

$$\mathrm{m} = -i\sigma_0\left\{\frac{\gamma}{\alpha - i\sigma_0}\int_{-\infty}^{t}\psi_{\mathrm{r}}\,dt + \left(1 - \frac{\gamma}{\alpha - i\sigma_0}\right)\exp\,[-(\alpha - i\sigma_0)t]\int_{-\infty}^{t}\right.$$
$$\left.\times\exp\,[(\alpha - i\sigma_0)t]\;\psi dt\right\}. \tag{12}$$

Combining this with equation (6) and noting that $\sigma_0 \gg \alpha \gg \gamma$, one obtains

$$\mathrm{m} = {}_0\dot{\psi}\left\{(t_0 t\gamma - \tfrac{1}{2}t_0^2\gamma + t_0) + \frac{i}{\sigma_0}[\![\exp\,[-(\alpha - i\sigma_0)t] - \exp\,[-(\alpha - i\sigma_0)(t - t_0]]\!]\right\}. \tag{13}$$

The first term of equation (13) is the secular trend in the motion of the pole while the second term is the wobble of the viscous body. In the limit, $\gamma \to 0$, $\alpha \to 0$, the solution reduces to that appropriate for an elastic Earth. In the absence of any information on the secular pole drift prior to about 1900, the viscoelastic solution (13) can only be tested against some 70 yr observations. With equations (4), (7) and (13)

$$\delta m_1 = 0.0342\gamma, \qquad \delta m_2 = -0.1039\gamma \tag{14}$$

for the change in **m** over the last 70 yr. The direction of the secular drift of the pole, which is independent of the value of viscosity, is

$$\tan^{-1}\frac{\delta m_2}{\delta m_1} = -72°, \tag{15}$$

in remarkably good agreement with the observed values summarized in Table 1.

Lambeck (1980) estimated that the direction of polar drift due to any present exchange of mass between the oceans and ice is of the order 30°–40° west but the Greenland ice sheet must be melting at an average rate of about 10 cm yr^{-1} to explain the magnitude of the observed drift. There is no evidence for or against this and the result suggests that there may be some contribution to the drift from present-day ice melting. Ignoring this complication, the result (15) can be used to infer the relaxation time and an effective Newtonian viscosity for the Earth. The isostatic factor l_s can be calculated from the relevant expression of Munk & MacDonald (1960) with the simplifying assumption that the Earth's crust rests on a homogeneous substratum, i.e.

$$l_s = 4\,\frac{H}{R}\left(1 - \frac{\rho_c}{\rho_e}\right), \tag{16}$$

where H and ρ_c are the average thickness and the density of the crust. The limiting values of l_s can be determined by considering two extreme cases: a homogeneous Earth with a mean density of upper mantle, and a homogeneous mantle with the mean density of the Earth. The two values for l_s are, with an average crustal thickness of 20 km 1.8×10^{-3} and 5.9×10^{-3} respectively. With a mean value of $l_s = 3.9 \times 10^{-3}$, equations (8) and (11) yield the range of relaxation times and viscosities indicated in Table 1 for a given set of observations of δm_1 and δm_2. The uncertainty in l_s should not affect the order of magnitude of viscosity given in Table 1. The average relaxation time and viscosity obtained from the first three rows of the table is $\sim 10^4(1 \pm 0.5)$ yr and $\tilde{\eta} = (1.1 \pm 0.6)\,10^{23}$ P. This viscosity is an effective viscosity of the Earth and, since it is determined from the global rotational response, it partly reflects the viscosity of the lower mantle. Since the post glacial studies by Cathles (1975) and Peltier & Andrews (1976) give lower viscosity values for the upper mantle, the above results indicate that there may be some increase in viscosity with depth.

Table 1. The relaxation time and viscosity of the Earth from astronomical observations of secular drift.

Observations*			Relaxation time τ (yr)	Viscosity $\tilde{\eta}$ (10^{23} P)
m_1 (10^{-8})	m_2 (10^{-8})	Source		
0.5670	−1.8545	Stoyko (1973)	9500 ~ 10200	1.0 ~ 1.1
0.5306	−1.4578	Styoko (1973)	10900 ~ 12100	1.1 ~ 1.3
0.7171	−1.5379	Markowitz (1968)	8100 ~ 11500	0.8 ~ 1.2
0.2272	−1.0421	Yumi & Wako (1968)	25500 ~ 16900	1.8 ~ 2.7

* See Lambeck (1980) for a further discussion of these observations.

5 Acceleration of the Earth's rotation due to deglaciation

The analysis of the secular drift of the pole given in the preceding section is based on two of the three equations of motion which involve m_1 and m_2. The third equation, expressing the change in the instantaneous rate of rotation, will also be affected by the viscoelastic response of the Earth to past deglaciation. The subject has been previously treated by Dicke (1969) and O'Connell (1971). These studies were based on a less detailed ice history and sea level changes than the ones now available. Also, recent analyses of the astronomical observations and tidal theory has resulted in revised estimates of the non-tidal acceleration of the Earth (Lambeck 1979, 1980).

The third equation of (3) for the viscoelastic Earth becomes

$$\dot{m}_3 = (1 + \hat{k}')\dot{\psi}_3$$

and with equation (9) for the Maxwell rheology

$$\dot{m}_3 = \frac{\hat{\mu} + l_s}{1 + \mu}\dot{\psi}_3 \simeq \frac{\hat{\mu}}{1 + \hat{\mu}}\dot{\psi}_3. \tag{17}$$

By substituting $\hat{\mu}$ and $\dot{\psi}_3$ from equations (1), (4) and (6) equation (17) becomes

$$\dot{m}_3 = \frac{\mu}{1 + \mu}\left(1 - \frac{\alpha}{D - \alpha}\right)\left[-\frac{\dot{c}_{33}}{C}H(t_0 - t)\right],$$

from which the $\dot{m}_3$ for the present time ($t > t_0$) follows as

$$\dot{m}_3 = \frac{\dot{c}_{33}}{C}\frac{\sigma_0}{\sigma_r}[\exp(\alpha t_0 - 1)]\exp(-\alpha t). \tag{18}$$

With the previously determined parameters

$$\dot{m}_3 = 0.083 \times 10^{-8}[\exp(-1.82/\tau) - \exp(-5.46/\tau)]$$

where τ is the relaxation time in units of 10^3 yr.

It is important to note that equation (18) is based on the assumption of a homogeneous Earth undergoing relaxation as a whole, including the core. Peltier (1974) has indicated that, for realistic Maxwell Earth models, the deviation from exponential relaxation is significant, especially for the lower degree harmonic components of the initial deformation. The most important reason for this is the gravitational attraction of the core which slows down the rebound and increases the relaxation time. Using the computations of Cathles (1975, pp. 90–99) we have estimated that the attraction of the core will lengthen the relaxation time by about 20 per cent. Introducing this revised τ, equation (18) becomes

$$\dot{m}_3 = 0.083 \times 10^{-8}[\exp(-1.52/\tau) - \exp(-3.03/\tau)]. \tag{19}$$

$\dot{m}_3$ relates to the secular acceleration $\dot{\omega}$ of the Earth according to

$$\dot{m}_3 = \frac{\dot{\omega}}{\omega}, \tag{20}$$

and Fig. 3 illustrates the variation of $\dot{\omega}/\omega$ with τ. As anticipated, the acceleration tends to zero for very small and very large values of τ. Lambeck (1979) has estimated a non-tidal acceleration of the Earth as $\dot{m}_3 = (0.69 \pm 0.26)10^{-10}\,\mathrm{yr}^{-1}$ on the assumption that the Newtonian gravitational constant G is time invariant. This yields a relaxation time centred around either 600 or 15 000 yr with a respective scatter of 500–700 yr and 11 000–

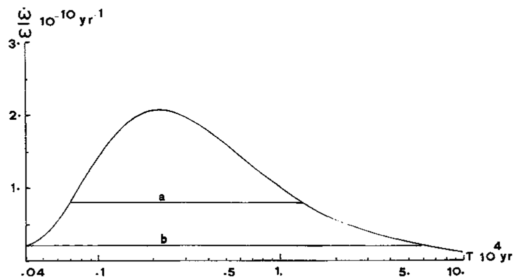

Figure 3. Non-tidal acceleration of the Earth as a function of relaxation time, (a) and (b) are the non-tidal accelerations for $\dot{G} = 0$ and $\dot{G} \neq 0$ respectively as estimated by Lambeck (1979).

27 000 yr. The effective viscosity deduced from these values is either 6.3×10^{21} or 1.6×10^{23} P, the latter estimate being in good agreement with that deduced from the polar drift. The uncertainty estimate of the larger value ranges from 1.2×10^{23} to 2.8×10^{23} P. The smaller value is close to that obtained by O'Connell (1971) although he did not consider the lengthening of the effective decay time due to the core effect. O'Connell's upper estimate was of the order of 100 000 yr. If $\dot{G} \neq 0$ the non-tidal acceleration is much reduced (Muller 1976; Lambeck 1979) and barely significant. This would argue for a much shorter relaxation time than the above estimates and is improbable. We conclude from the analysis of both polar motion and length of day that the effective mantle viscosity lies in the range $(1-2) \times 10^{23}$ P.

6 Comments

The above results indicate that the present secular drift of the pole can be attributed to the Earth's ongoing response to the deglaciation during the period 18KYBP–6KYBP. Since the estimated direction of drift is independent of the Earth's viscosity, any small changes in ice-melting history and the isostatic factor will not effect this conclusion.

Accepting this interpretation, the observed rate of the secular drift of the pole as well as the non-tidal acceleration of the Earth suggest an effective mantle viscosity of between $(0.7-1.6) \times 10^{23}$ P. If the melting of the ice sheets occurred in a shorter time than assumed above and was complete before 6KYBP, this would result in increased viscosity. Hence, although the melting regime has significant bearing on the computed magnitude of viscosity, the viscosity deduced here should possibly be considered as a lower bound.

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