Geodesy and Artificial Earth Satellites

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One of the fundamental aims of geodesy is the establishment of a precise and consistent reference system that embraces the entire world. Such a system would consist of two closely related parts; the geometric elements which specify the relative positions of any points in the system, and the dynamic elements which everywhere define the direction and magnitude of gravity. These two groups of elements may theoretically be considered as independent, but, in their derivation and application, the distinction is not as clear cut. For example, terrestrial triangulation can, theoretically at least, be achieved without any direct or implied reference to the direction of gravity if vertical angles between the observing stations are measured accurately. But in practice this approach breaks down for obvious reasons and dynamic quantities must be introduced instead if the co-ordinate system is to be adequately established.

The purposes of a consistent world co-ordinate system are manifold; both practical and scientific, though it is perhaps the latter that offers greater excitement. Geophysical theories such as continental drift, heat flows in the earth's interior, movements of the earth's axis of rotation within the crust and earth tides have too often in the past relied on very indirect evidence for their verification or rejection and these studies would benefit tremendously if the phenomena could be observed directly. But this can only be done if a unified reference system exists.

Space scientists require precise tracking of many of their satellites and space vehicles in order to correlate measurements made by their instruments with position in space. Satellite-borne cameras for synoptic world weather studies, reconnaissance and astronomical studies require, for example, a precise knowledge of the positions and orientations of the space craft and this, in turn, demands a detailed knowledge of the tracking station positions and of the earth's gravity field extended outwards into space.

Navigation can also be achieved with the aid of satellites—one system is already in operation by the United States Navy—once the forces acting on satellites become predictable and the locations of the ground monitoring stations are accurately known.

The problem of obtaining a world wide co-ordinate system has only recently appeared to be capable of solution. Existing triangulation or traverse schemes suffer from the handicap of being subject to irregularities in the direction of the vertical at the observing stations. This would be no problem if these directions could be accurately related, but the present inaccuracies of gravity and vertical angle measurements make any such relations of dubious practical value. Instead the computations are usually carried out over mathematical surfaces, whose relation with the physical surface is not always accurately defined.

The curvature of the earth also limits the lengths of the sides of triangulations and traverses, resulting in unfavourable error propagation when very large areas are considered. The relative positions of nearby points will, as a rule, still be accurately defined but the relative positions of distant points become uncertain. But often these are just the points in which the earth and space scientist is most interested. Of importance to him are the positions of points separated by very large distances.

Then there is the problem of bridging oceans so that the various land masses and islands can be connected into a uniform system. Obviously conventional geodetic measurements cannot be used here and systems such as Hiran and flare triangulation can only be used to a limited extent when the areas to be bridged are separated by hundreds rather than thousands of kilometers.

Astronomical observations, coupled with gravity measurements, have been suggested as a method of establishing a world reference net but, while such a system is in most cases adequate for mapping purposes, it is insufficiently precise for many geophysical investigations.

These remarks are of course not meant to infer that the existing and established methods of geodesy are of little value. Quite the contrary, only they do not offer the means of accomplishing the desired world net.

Artificial earth satellites have not offered a new approach to solving the basic problem of geodesy but they have provided new tools, with which a precise world reference system may be established. This is a somewhat paradoxial situation; the advent of the space age provided an urgent impetus for establishing a precise world reference system, yet this system can only be established using the vehicles of this new era. The earth's only natural satellite, the moon, has been used in the past in attempts to determine the world system but its great distance from the earth and its relatively large size have ensured that only approximate solutions have been possible.

The considerable height of the satellite makes it possible for very much greater distances—of the order of thousands of kilometres—to be bridged now, and the motion of the space vehicle along its orbit enables large areas of the earth's gravitational field to be sampled within a relatively short time interval. There are therefore two ways in which the satellite can be used in geodesy. It can be considered purely as an elevated point in space and used to determine the relative station positions by a process of intersection and resection. Alternatively or concurrently, it can be used as a gravimeter; the perturbations in the orbit resulting from gravity anomalies are used to interpret the shapes of equipotential surfaces in space.

The two satellite methods are essentially independent, yet complementary. One provides the geometric reference system and the other provides the dynamic system. Obviously the satellite should be used

Note: This sheet is to be inserted between pages 114 and 115

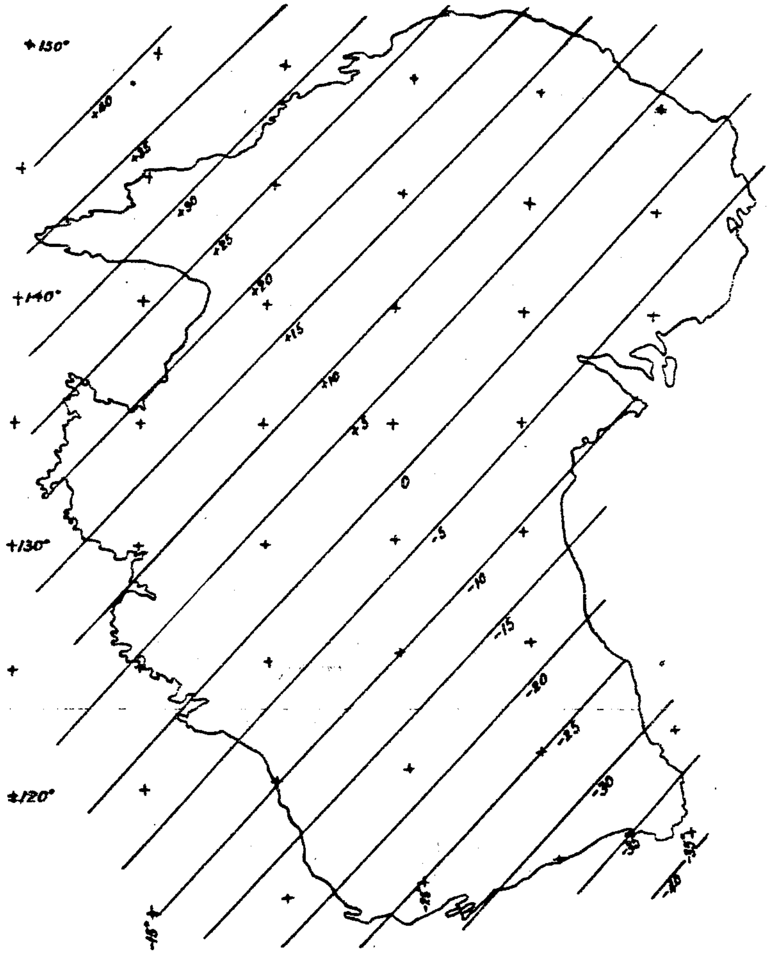

Fig. 1. Height Differences between S.A.O. Reference System (6 378 160 m. and 298.25) and Australian National Datum (Johnston Origin, 6 378 160 m. and 297.25)

to determine positions only for points separated by large distances and they can be used to provide only a generalized picture of the equipotential surfaces. Yet there are just the parameters that the conventional methods cannot provide. Satellite geodesy therefore does not supplant the terrestrial methods; the two will work in complete harmony, the former giving a precise framework that is consistent for the entire world and the latter providing the detail between the satellite control points.

This is the way in which it is envisaged that the satellites will help the geodesist and the results of the last few years indicate that these aims are being achieved already.

Thus there are three aspects of establishing the world net, namely the geometric and the dynamic satellite solutions, and the terrestrial solution. The first two provide the generalized shape of the earth while the last provides the more detailed data, which is linked with the first two.

The geometric satellite solution has the characteristic that it is much less dependent than the other two methods on a multitude of inherent assumptions. It may therefore be considered as the basic framework and the ideal solution will be one that first establishes the geometric frame. The results would then be used in the dynamic solution to determine the station positions with respect to the earth's centre of mass and to determine the parameters defining the gravity field. Only when both these systems are established, should the terrestrial data be linked up with the functional whole. But the urgency, with which the various aspects of the different solutions are required, will often rule out such an orderly approach, at least for the present. Yet it should provide the guide lines for future developments.

At the present stage of the growth of satellite geodesy, operational aspects and research just cannot be kept isolated as separate entities. On the contrary, they are very closely related. Little is known at present about the inherent accuracies of the observational processes used. Unlike many terrestrial geodetic measurements there are seldom ready made rules available which are based on experience and which can be used for assessing the reliability of measurements. Neither has the question of the necessary and desirable distributions of observations been thoroughly investigated. Even the nature of error propagation plays a quite different role in satellite geodesy from that in most terrestrial measurements, where a greater control over environment is feasible. Observations will be time dependent and systematic errors will tend to dominate the random ones, yet often the presence of the systematic errors can only be suspected with the overall result that the calculus of observations and the adjustment techniques may need a new approach.

The above comments have briefly outlined the purposes, the methods and the problems of satellite geodesy. Of necessity they have been brief and, for equally good reasons, they have not always been positive but they do indicate the ideas behind satellite geodesy and they do indicate the possibilities of realizing a uniform world geodetic system. Moreover, what is important is that this unified world system can be established in a relatively short time space, of the order of a few years. The next step is therefore to discuss in more detail the actual methods employed. Before this can be done however, some remarks must be made about the observational procedures, for no theory, however sophisticated, is of practical value if it is not compatible with the observational material available.

SATELLITE OBSERVATIONS FOR GEODESY

Observed quantities which determine the satellite positions relative to the tracking stations are directions, ranges, range rates or any combination of these quantities. For the geometric method the first two types of observations are of most value for the dynamic solution all types of measurements are important. In addition to these position observations, the instant of observation is required to a high degree of accuracy—of the order of a mille-second or less—since the satellite will move relative to the observer with velocities of up to about 10km/sec.

Direction observations can be made in a variety of ways, using either optical or electronic principles. The optical methods may be either visual, photo-visual or photographic but it is only the last of these—combined with precision photogrammetric or astrometric reduction techniques—that gives accuracies of geodetic significance and directions that refer to a consistent stellar framework rather than a local or some intermediate reference system.

The instrumentation developed for photographic observations is extremely varied. This includes tracking cameras such as the Baker-Nunn and the Dual Rate camera of Markowitz, cameras driven at sidereal rates such as the MOTS or stationary cameras such as the Wild BC4 and the Hewitt camera. But in essence, they all furnish the same information, a photographic record of the satellite trail chopped into a number of short segments by a rotating shutter, a number of star images, which provide the reference points and the time records at which the satellite trails have been interrupted and at which the star exposures have been made. With this information the satellite position may be determined in the astronomical system at a known time instant, namely the declination and right ascension of the satellite at the instant of observation.

The accuracy with which a single direction to a satellite can be determined is at best of the order 1″ while more typical values would be from 1.5″ to 2.0″ and it does not appear that any significant increase in these accuracies are presently attainable. The chief sources of uncertainty are errors introduced by the time measurements, irregularities in atmospheric refraction or shimmer, plate measuring errors and uncertainties in the reference star catalogues used. The motion of the satellite relative to the observer does not permit a repetition of the observation under the same circumstances and the only means of improving the accuracy of a direction in space is by curve fitting and interpolation techniques. Thus, if a sequence of satellite images are recorded at short intervals, a polynomial can be fitted through these positions and, by interpolating for an arbitrary instant, a fictitious satellite position, which has a higher accuracy than the individual observations can be computed. The accuracy attainable for the fictitious positions is of the order 0.5″ to 1.0″; and the first value probably represents the upper limit of attainable direction accuracies.

Interferometer techniques, using micro-waves, provide a most useful method of making direction observations if a large number of semi-precise observations are preferable to the fewer—more precise—measurements that are obtainable with optical tracking. A serious shortcoming of these observations is their low accuracy, of the order of several minutes of arc, due to ionospheric refraction. Consequently, they are of little importance for precision geodesy though in all fairness it must be remembered that they provided most useful information in the early days of satellite geodesy.

Ranges to satellites can be measured using either micro-wave frequencies or optical frequencies. The latter, in the form of pulsed lasers, is particularly important as it offers one of the most precise methods of satellite tracking. The laser is used essentially as a radar; the laser beam is directed towards a satellite—which may or may not carry special retro-reflectors—and the echo is received by a detector back on the earth's surface. Lasers have an advantage over conventional radar because very much more energy is contained in a single laser pulse and a more refined collimation of the beam is possible without requiring the huge antennae, that are characteristic of most radar ranging methods. In addition, the accuracies attainable are much greater for laser ranging than for radar.

As well as using the laser pulses for ranging, another obvious use is to employ them as searchlights to illuminate satellites when they are in the earth's shadow. The greater penetrating power of a laser makes it a much more powerful tool for this purpose than conventional searchlights. The object illuminated in this manner could then be photographed against the star background and its direction in space determined.

The ideal solution would be to determine the directions and ranges simultaneously by photographing the momentarily illuminated satellite against the star background as well as measuring the time interval that the pulse takes to travel from the transmitter to the satellite and back to the receiver. This has indeed been tried but the results are inconclusive on account of the conflicting demands that the two types of measurements placed on the pulse duration. Precise range measurements require pulse durations of about 10 to 20 nano-seconds, while photographing the laser illuminated satellite requires pulse durations of the order of 1 to 2 mille-seconds.

The accuracy of a single laser range measurement—scaled to the velocity of light—is presently of the order of 1 to 1.5 meters for satellite ranges of 1,000 km or more. Accuracies of 20 to 30 cm are, however, feasible with minor modifications of the present instrumentation. This is considerably higher than the direction observations and the only way of efficiently combining direction and range measurements is to observe the satellite optically over a short arc which embraces the range measurement and to interpolate for the position that the satellite occupied at the instant in which the range is observed.

Of the several possibilities of ranging to satellites using microwaves the most suitable appears to be a phase comparison system allied with special transponder carrying satellites. Objects in orbit carrying phase transponders include the geodetic satellites ANNA and GEOS and the special SECOR satellites—the EGRS series.

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The phase comparison method is similar to that of the tellurometer, the remote being carried in the satellite while the master is ground based. The SECOR system as used by the United States Army is essentially a method of trilateration in space; four ground stations measuring the ranges to the satellite "simultaneously".

The accuracy of a single micro-wave range measurement is of the order of 3 to 4 meters in the zenith and decreases with increasing zenith distance according to a secant rule. The principal uncertainties are due to ionospheric refraction and the calibration of the system.

But the significant distinction, however, between the laser and micro-wave methods of ranging is not so much the difference in their uncertainities as the nature of the errors contributing to the total amount. For laser ranging all the important error sources tend to be random from measurement to measurement while the micro-wave observations are seriously subjected to systematic or model errors which may persist for the entire satellite pass or for even longer periods. In this latter case, the repetition of observations only improves the consistency but it does not increase the reliability of the observations.

METHODS OF SATELLITE GEODESY

(i) The spatial triangulation method

Photographing an illuminated object against the star background defines the object's position uniquely in the stellar framework. Observing the object simultaneously from two stations defines a plane in space. Repeating the observation when the object is in a different position defines a second plane, the intersection of which with the first plane gives the direction of the line joining the two stations in the same reference system as the star positions and if the relationship between this framework and the terrestrial system—defined by the pole and the Greenwich meridian at a given epoch—is known, the direction between the two observations is fixed with respect to the earth.

With a lot of care and a bit of good luck directional accuracies of 0.5″ or possibly even 0.3″ can be obtained for the vectors joining participating stations.

By observing to more objects and incorporating more stations into the scheme a network of triangles connecting the observatories can be established. The orientation of the scheme is fixed by the star catalogue used but the scale and origin are arbitrary because of the infinite distance to the stars, and they must be determined by other techniques.

The principle of spatial triangulation is by no means a new concept. It was applied to shell and rocket trajectory measurements in the 1930's by Hoppman and Lohman and again suggested by Vaisala in 1946 using flares ejected from balloons. Consequently it has been used for ballistic missile tracking in the United States but it was the advent of artificial satellites that gave a new impetus to use the method in geodesy. Principal in the recent progress of satellite triangulation are the Smithsonian Astrophysical "Observatory's Baker-Nunn tracking net, the United States Coast and Geodetic Survey's North America and world projects and the Institut Geographique National's France-North Africa project.

Characteristic of the spatial triangulation method is its complete independence of the direction of the vertical; all observations are related to a consistent astronomical system and free from the problems of relating the directions of the vertical at the different stations. Furthermore, because lines of sight are well elevated, atmospheric refraction (both vertical and lateral) plays a lesser role than in terrestrial geodesy, thus avoiding the rather artificial separation into planimetry and height that is often necessary in terrestrial geodesy. The distance between adjacent stations is now no longer limited by intervisibility but is essentially a function of the satellite height. Fewer "primary" or "basic" triangles are therefore required to cover any given area with the consequence that there is an important reduction in the propagation of the variances of the observations and parameters through the triangulation scheme.

Several possibilities suggest themselves for scaling the triangulation. One is from terrestrial geodetic measurements. This requires some knowledge of gravity along the base line in order to reduce the distance to the reference ellipsoid and hence to transform it into a chord length. Another method is to range to the satellite, using either laser or microwave methods, at the same time as the simultaneous direction observations are made. Laser measurements are particularly promising in this context and some initial considerations have shown that this is the most suitable method for introducing scale into the triangulation, and that the magnitude of the vector joining stations—scaled again to the velocity of light—can be determined to within 1 in 500,000 or possibly even 1 in 700,000. Yet another method is to use the SECOR system, but the problem with this system is that the signals are subject to irregularities in ionospheric refraction and that the instrumentation suffers from some serious uncertainities.

The translation of the oriented and scaled system can be fixed by adopting some arbitrary datum but it would be preferable if it could be centred at the earth's centre of mass. Dynamic satelite methods make such a geocentric system possible.

(ii) The dynamic solution

If the earth were a homogeneous sphere, orbiting satellites would obey Keplerian motion and the satellite motion could be defined by six constants which define the size, shape and orientation of the trajectory in space. Once these constants are known the satellite position is uniquely specified at any instant by simple equations of celestial mechanics.

Obviously the earth is not so obliging as to be able to be approximated by a mass point, but this illusion can be retained a little longer for illustrative purposes. Denote the station position relative to the earth's centre of mass by the vector $\vec{R}$, the satellite position relative to the centre of mass by the vector $\vec{r}$ and the topocentric position vector of the satellite by $\vec{\rho}$. Then these three vectors are related by the simple vector equation—

$$\vec{R} + \vec{\rho} = \vec{r}.$$

If the station position is known and directions to the satellite are observed the above expression represents three equations in four unknowns; the magnitude of the vector $\vec{\rho}$ and the three components of $\vec{r}$. But these three position components are related to the orbital elements by simple transformation formulae so that three sets of direction observations suffice to determine the six constants and the three distances to the observed satellite positions.

Consider now that the same object is observed from a station whose position is unknown. For the instant of observation the satellite's geocentric position vector can be computed from the orbital elements and each set of observations produces a set of equations, like the above, from which the station position can be determined if at least two sets of direction observations are made. In practice neither the station positions nor the satellite's orbital elements are known initially and an iterative solution is required. From approximate station positions approximate orbital elements are computed which are in turn used to improve the values for the station positions and the cycle is repeated until a satisfactory convergence has been reached.

But the above description of the dynamic method is a very gross simplification of reality. The irregular mass distribution of the real earth and other—both gravitational and non-gravitational—forces ensure that the Keplerian motion is a most inadequate approximation. The effect of the perturbing forces is such that the orbital constants are no longer constant but that the orbit's shape, size and orientation undergo perturbations, some of which may be of a secular nature and others periodic. Theoretical expressions for these perturbations can be derived by expanding the earth's potential into a series of zonal and tesseral harmonics and substituting these expansions, for example, into the Lagrangian equations of planetary motion. By observing the actual motion, filtering out the effects of the other perturbing forces, and comparing with the theoretical expressions it becomes possible to find a solution for a number of the harmonics and consequently the shapes of the equipotential surfaces. In a complete solution, therefore, parameters to be solved for include the orbital elements of the satellite arcs observed, the coefficients defining the potential field, the station positions and any others necessary to define the other perturbing forces such as air drag and solar radiation. This becomes an immense task requiring thousands of observations of different satellites taken from numerous stations distributed around the globe over periods of several years.

Some Results

One of the most recent and reliable dynamic solutions has been determined by the Smithsonian Astrophysical Observatory from thousands of Baker-Nunn observations made over a period of about seven years. The solution includes the effects of the zonal harmonics up to degree 14 and the tesseral harmonics up to degree and order 8. The station positions have also been determined to an accuracy, relative to the centre of mass, of about 10 to 15 meters. This particular solution is called the 1966 Smithsonian Institution Standard Earth. From this geoid the parameters for a best fitting ellipsoid can be deduced. They are—

$$a = 6\,378\,165 \text{ m},$$

$$1/f = 298.25$$

and on this reference ellipsoid the value for the gravity at the equator is

$$g_e = 978.0306 \text{ gal.}$$

which corresponds to the adopted value of GM

$$GM = 0.7537172 \times 10^5 \text{ Mm}^3 \text{ day}^{-2} \text{ rev}^2.$$

To this ellipsoid the existing geodetic datums can be connected. For example, the Australian Datum (Johnston origin) connected to the SAO Standard Earth gives the results indicated in figures 1 to 3. The tilt of the Australian Datum with respect to the Standard Earth is commensurate with the geoidal slope across the Australian Continent.

Some initial calculations for an Australasian satellite triangulation scheme have been carried out by the writer. The scale is introduced from simultaneous direction and laser range measurements. One of the outcomes of this study is that with the satellites presently in orbit, the time required to determine the distance between two stations is very much longer than the direction determination, by a factor of about three or even four. Thus while it is desirable, from a propagation of variances viewpoint, to introduce as many scale determinations as possible, this is not a practical approach. If only one scale is determined then the maximum overall accuracy attainable is of the order 2 in 1,000,000 for stations separated by distances of up to 4,000 km. The introduction of two additional scales gives an overall accuracy of the order 1.5 in 1,000,000 and this is probably the most economical solution. If three cameras such as the SAO K-50 were available and used in conjunction with the Baker-Nunn camera at Woomera, the total observing time for the directions will be of the order of three years and if two lasers are available a further three years are required to scale the net. These time intervals are based on the suitable satellites presently in orbit and they will be considerably reduced in the future when more objects are in orbit. The results however suffice to show that satellite triangulation is feasible and that it will give highly accurate results in a relatively short time interval. The details of the study have been presented elsewhere.

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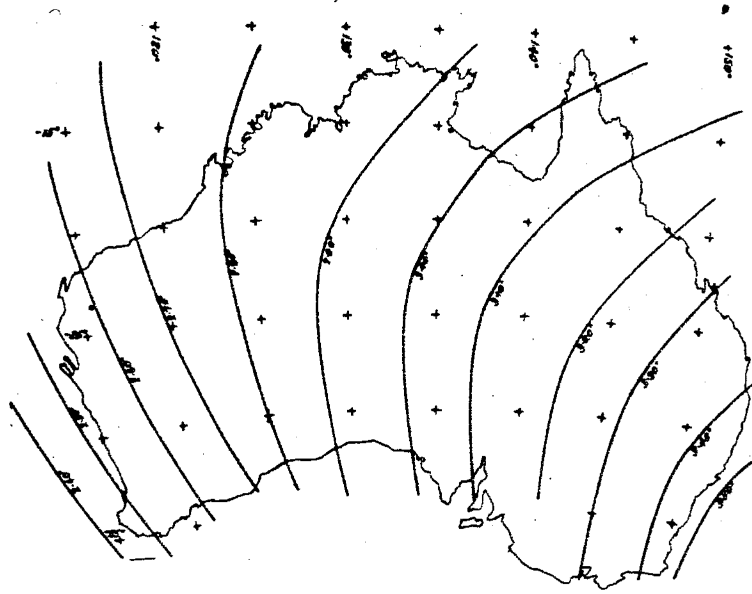

Fig. 2. Deflections of the Vertical between S.A.O. Reference System and A.N.D. – Component in Meridian.

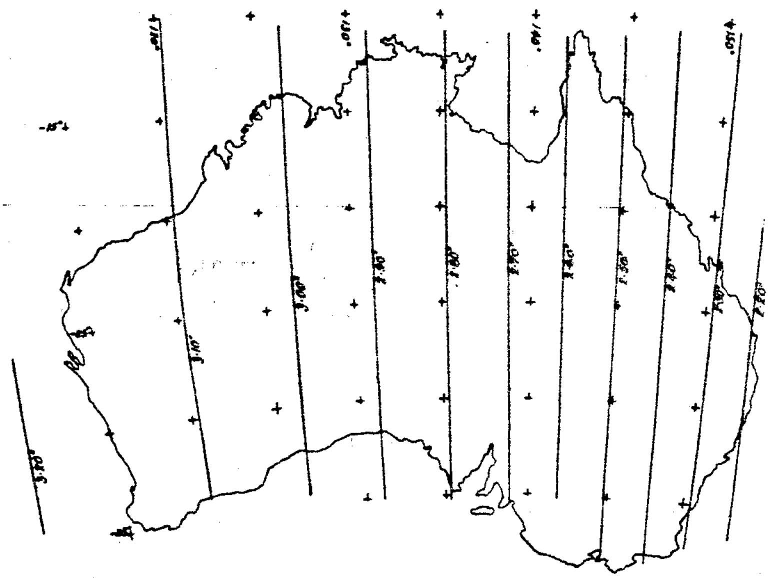

Fig. 3. Deflections of the Vertical between S.A.O. Reference System and A.N.D. – Component in Prime Vertical.