Reprinted from the

PROCEEDINGS OF THE TWELFTH LUNAR AND PLANETARY SCIENCE CONFERENCE

Houston, Texas, March 16–20, 1981

Pergamon Press

New York • Oxford • Toronto • Sydney • Frankfurt • Paris

Proc. Lunar Planet. Sci., 12B (1981), p. 853–865.
Printed in the United States of America

Mascons and loading of the lunar lithosphere

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Abstract—In the usual applications of elastic plate theory to mascon loading problems, a number of assumptions have been made whose consequences on the final results have not been fully investigated. We examine some of these assumptions here. Two modifications of the elastic thin plate theory have been made to account for the depth dependence of the rheology and for any lateral variation in the flexural ridigity of the lithosphere. Viscoelastic and elastic-viscoelastic layers over an inviscid media have also been considered. These modifications can significantly perturb the surface stress distribution from that predicted by the thin plate theory. The use of the latter for estimating the thickness of the lithosphere, by relating predictions of the stress field to surface evidence of stress, can lead to significant underestimation of the real thickness of the outer stress-bearing layer of the moon.

INTRODUCTION

Muller and Sjogren's (1968) discovery of the large positive gravity anomalies over the circular mare raised at least three important questions associated with the implied mass concentrations or mascons: the nature of the subsurface structure, the nature of their long-term support and the mechanism of formation. Despite having been subject to much investigation these questions have remained largely unresolved (see Phillips and Lambeck, 1980). Until recently, the effort has been concentrated on establishing plausible density models in accordance with the observed gravity anomalies. These models have evolved from ones where the excess mass is attributed entirely to the basaltic lava fill (e.g., Conel and Holstrom, 1968; Baldwin, 1968; Kunze, 1974) to ones where the dominant contribution has come from the deformation of the crust-mantle boundary associated with massive disruption of the lithosphere by the basin-forming impact (e.g., Wise and Yates, 1970; Bowin *et al.*, 1975; Sellers, 1979). The former models require very thick mare basalts in the circular basins, of the order of tens of kilometers, but according to several lines of evidence this is unsatisfactory. Studies of partially flooded craters in mare basins have indicated that, in general, basalt thicknesses are of the order 1–2 km or less (DeHon, 1979; and earlier papers). In some areas, these estimates have been corroborated by the presence of highland material in the ejecta of post-mare craters. Thus the presently accepted density models associated with lunar mascons are all based on the existence of a substantial mantle plug beneath the mare.

Tectonic features are not a prominent characteristic of the lunar surface but those that do exist tend to be associated with mare basins. This relation was noted by Phillips *et al.* (1972) who suggested that the tectonics were associated with the brittle failure of the crust subjected to the mass excess. More recently Melosh (1978) calculated the surface stresses for a simple mascon model consisting of an axially symmetric normal load on an elastic lithosphere overlying an inviscid fluid. Melosh noted that the radial dependence of the surface stress-state was a sensitive function of the lithospheric thickness. Solomon and Head (1979, 1980) took this further by interpreting the concentric rilles observed around circular mare basins as graben features, and using their spatial distribution to estimate the thickness H of the elastic lithosphere as a function of space and time. Normal faulting occurs when the vertical stress σ_{zz} is the maximum principal compressive stress, and the failure plane will contain the intermediate principal axis. If the radial stress

σ_{rr} is the maximum tensional stress then concentric grabens could form. As pointed out by Melosh, such a stress-state exists at a distance r from the centre of the load of approximately four times the flexural parameter ℓ (defined by Eq. 2 below). But larger stress differences occur closer in, and failure by strike-slip faulting would be expected in the approximate distance range $2\ell \lesssim r \lesssim 4\ell$, or by radial thrustfaulting at $r \lesssim 2\ell$. Solomon and Head proposed that during the early basin filling phases, a global thermal stress increased the radial (σ_{rr}) and tangential ($\sigma_{\theta\theta}$) stresses to the point where $\sigma_{\theta\theta}$ was the intermediate principal stress and σ_{rr} and $\sigma_{\theta\theta}$ were both tensional in the distance range $r \gtrsim 2\ell$. Thus if failure occurred it would have been normal faulting. On this basis Solomon and Head concluded that the lithosphere was relatively thin ($25 \leq H \leq 75$ km) 3.6–3.8 b.y. ago and that this thickness increased to about 100 km in the interval 3.6–3.0 b.y. ago. These conclusions have important implications regarding the thermal state of the moon 3–4 b.y. ago as well as on the question of the stress-bearing capability of the moon as a function of depth.

The flexure theory used by Solomon and Head is, however, based on a simple and highly idealized model, and consideration has to be given to several factors before these results can be assumed to be definitive. The purpose of this paper is, therefore, to investigate some of these factors. The emphasis at this point is on the mathematical models rather than on their application to a particular mascon and we leave aside the question of the formation of the graben. Flexure models that have been investigated include:

1. Elastic shell models whose properties vary with depth.
2. Elastic shell models whose flexural rigidity is a function of position so as to investigate the response of a shell that has been thinned or weakened in the central region.
3. Viscoelastic plates.
4. Elastic layers over a thick viscoelastic layer.

ELASTIC PLATE THEORY

The theory of the deflection of elastic plates over a fluid substratum due to normal surface loads is discussed in many mechanical engineering texts (e.g., Nadai, 1963; Szilard, 1974). A theory frequently used in geophysics is for thin plates with small deflections where the governing equation is

$$\nabla^4 w + \frac{w}{\ell^4} + \frac{q}{D} \tag{1}$$

where w is the deflection (positive downwards), q is the normal load, D is the effective flexural rigidity of the plate, and ∇^2 is the Laplacian operator. The parameter ℓ, with the dimension of length, is defined as

$$\ell^4 = \frac{D}{\rho_s g} \tag{2a}$$

where ρ_s is the density of the substratum and g is gravity. This definition for 1 is valid for a flat plate. For a shallow spherical shell of radius R, rigidity μ and Poisson's ratio ν,

$$\ell^4 = D[\rho_s g + 2\mu H(1+\nu)/R^2]^{-1} \tag{2b}$$

where H is the effective elastic thickness of the shell (Reissner, 1946; Nadai, 1963; Brotchie, 1971). For a homogeneous elastic plate or shell

$$D = \frac{\mu H^3}{6(1-\nu)}. \tag{3}$$

For a disc load of radius A, the solution of Eq. (1) may be expressed in terms of

Kelvin-Bessel functions of zero order. That is, with $x = r/l$, $a = A/l$

$$w(r) = \frac{q\ell^4}{D}[1 + a\,\mathrm{ker}'a\,\mathrm{ber}\,x - a\,\mathrm{kei}'a\,\mathrm{bei}\,x] \qquad x \leq a$$

and

$$w(r) = \frac{q\ell^4}{D}[a\,\mathrm{ber}'a\,\mathrm{ker}\,x - a\,\mathrm{bei}'a\,\mathrm{kei}\,x] \qquad x \geq a. \tag{4}$$

For radially symmetric loading the stresses within the plate are

$$\sigma_{rr}(r, z) = \frac{-12D}{H^3} z\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right)$$

$$\sigma_{\theta\theta}(r, z) = \frac{-12D}{H^3} z\left(\nu\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right) \tag{5}$$

where z is the distance (positive downwards) from the middle plane. The components of shear stress are generally small so that σ_{rr}, $\sigma_{\theta\theta}$ and σ_{zz} are approximately principal stresses. At the outer surface, the principal stresses are given by $\sigma_{rr}(r, -H/2)$, $\sigma_{\theta\theta}(r, -H/2)$ and $\sigma_{zz}(r, -H/2) = q(r)$.

This thin plate theory is based on the assumptions that w/H and dw/dr are small quantities, that stretching of the middle plane during deformation is zero, and that in-plane forces acting on the middle plane are zero. In the application of this theory to the mascons, the first condition is generally satisfied but the other assumptions may not be valid if H is large. From a comparison of the simple theory with the thick-plate theory given by Frederick (1956) we have concluded that the former is quite adequate if $H \leq 100$ km for typical mascon loads, and for rigidities $\mu \gtrsim 5 \times 10^{10}$ dyne-cm^{-2}. Other obvious assumptions are that (1) at some depth H the lunar mantle becomes sufficiently weak for it to be treated as inviscid fluid when subjected to stress differences on a time scale of 10^9years and (2) the lithosphere acts as a homogeneous and competent layer whose properties are unaffected by the process that formed the mascon. The validity of these assumptions is perhaps less obvious.

Figure 1 illustrates the response of an elastic shell subject to an axisymmetric surface load of two superimposed discs whose properties are summarized in Table 1. This load geometry is used in all examples discussed. The response is dependent on D which is strongly dependent on H and, to a lesser degree, on the elastic properties of the shell (Eq. 3). Deflections, stresses and the stress difference $\sigma_{rr} - \sigma_{\theta\theta}$ are given for several values of H and the results clearly indicate sensitivity of the location of maximum stress at the surface to the thickness of the shell. The tectonic regimes corresponding to the surface stress state of this model have been discussed by Melosh (1978). If stress differences at the surface of the moon exceed the finite strength within the mascon basin, then failure would result in radial thrust faults. Outside the loaded area any failure would result first in strike-slip faulting, and only beyond the point where $\sigma_{\theta\theta}$ becomes tensional would normal faulting be predicted. The magnitude of the stress-differences reaches a maximum in the distance range of 200–300 km and this is relatively insensitive to the lithosphere thickness. Hence if failure occurred beyond the mare basin the dominant faulting would be strike-slip. There is no evidence of this and, always assuming that the flexure model is valid, either failure has not occurred or a regional tensional stress must be introduced such that $\sigma_{\theta\theta}$ becomes negative and is then the intermediate principal stress. Failure would now be possible by concentric normal faulting, but it should be noted that the required additional tensional stress is of the order of at least a couple of hundred bars and not particularly modest as indicated by Solomon and Head (1979, page 1673).

The lunar lithosphere is unlikely to be homogeneous in so far as its elastic parameters are concerned. Thus D and H must be considered as effective parameters. For an elastic layer with depth dependent elastic moduli, the effective flexural rigidity may be defined as

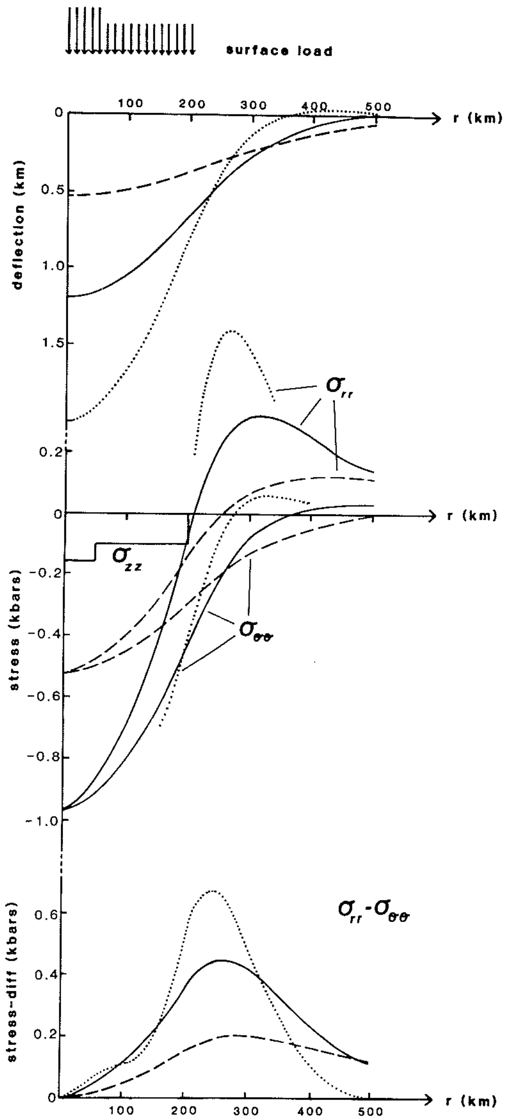

Fig. 1. Deflection, stresses and the stress-difference $\sigma_{rr}-\sigma_{\theta\theta}$ at the surface of an elastic shell (radius = 1738 km) of rigidity $\mu = 0.3\times10^{12}$ dynes/cm^2 and Poisson's ratio $\nu =$ 0.30, overlying a fluid substratum of density $\rho_s = 3.4$ g/cc. The applied load is given in Table 1. Results for shells of different thicknesses are illustrated: dotted lines correspond to H = 25 km; solid lines, H = 50 km; dashed lines, H = 100 km.

a depth-integrated function of μ and ν. That is (Szilard, 1974, p. 391; Lambeck and Nakiboglu, 1981):

$$D_{eff} = B_3 - B_2^2/B_1$$

where

$$B_i = 2\int_0^H \frac{\mu(z)}{1-\nu(z)} z^{i-1}\, dz \qquad (i = 1, 2, 3).$$

In the integral z is measured downwards from the top of the plate. The deflection follows

Table 1. Parameters of the applied surface load.

	A(km)	ρ_ℓ(g/cc)	h_ℓ(km)	q(bars)
disc 1	200	3.2	2	104
disc 2	50	3.2	1	52

from Eq. (4) with D replaced by D_{eff}. The stress components are

$$\sigma_{rr}(r, \xi) = \frac{-2\mu(\xi)}{1-\nu(\xi)}\xi\left(\frac{d^2w}{dr^2} + \frac{\nu(\xi)}{r}\frac{dw}{dr}\right)$$

$$\sigma_{\theta\theta}(r, \xi) = \frac{-2\mu(\xi)}{1-\nu(\xi)}\xi\left(\nu(\xi)\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right) \tag{6}$$

where ξ is the distance from the neutral plane which is located at a depth $z = B_2/B_1$ below the upper surface.

Figure 2a illustrates some examples of the response of a loaded laminated shell. In model A (see Fig. 2b) the layers decrease in strength with depth, while in model B the rigidity at first increases with depth so the maximum strength occurs at some depth below the surface, and then decreases. The first situation can be anticipated from the increase in temperature with depth, while the second situation would arise if the upper layers of the crust are weak due to low confining pressures. The stresses are now concentrated in the stronger layers and the large stress differences at the base of the plate, consequential to the simple plate models, are avoided. This is further illustrated in Fig. 3 where σ_{rr} is plotted as a function of depth for a 50 km thick homogeneous elastic shell, and for a shell characterized by the same flexural rigidity in which μ varies with depth according to a cosine function. These models indicate that a literal interpretation of the results from the simple plate theory leads to an underestimation of the thickness of the stress-bearing layer by an amount that will be a function of the actual rheology of the lithosphere. That is, the lithospheric thickness estimates of Solomon and Head are effective or equivalent thicknesses only.

One of the limitations of the elastic plate theory as used above is that it totally ignores any effect of the basin-forming impact on the lithospheric structure since the load is assumed to rest on a competent plate. But the catastrophic impact events would have led to major disruptions of the crustal structure. This is implied by the second class of gravity mascon models in which upwelling of the mantle material occurred in response to the crater excavation. This upwelling, together with the dissipation of impact-generated heat and the brecciation of material by shock waves, would have resulted in a significant thinning and weakening of the crust below the basin. As the moon cooled, the topography of the mantle-crust boundary would have been frozen into the upper mantle. The resulting mantle plug and the subsequent mare basalts then form the major contributions to the mascon. This plug may have two consequences for the flexure model: it introduces lateral variations in the properties of the plate and it introduces a pre-stress in the plate corresponding to the state of isostasy prior to the mare fill-in.

To estimate the effect of lateral variations in the plate, we have adopted a model in which the effective flexural rigidity is assumed to be discontinuous at $r = r^*$ such that $D = D_1$ for $r < r^*$ and $D = D_2$ for $r > r^*$. This discontinuity can be interpreted as either a change in the effective thickness of the plate or as a change in its elastic properties. Figure 4 is a schematic illustration of the model, its mathematical approximation and its mathematical equivalent. Equation (1) applies to each region separately and the two solutions are matched by imposing the condition of continuity of w and its derivatives (dw/dr, d^2w/dr^2, $d\nabla^2w/dr$) at $r = r^*$. The solution is similar to that for seamounts discussed by Lambeck (1981). Once w is determined, the stress components follow from Eq. (5). Results for this model are illustrated in Fig. 5b where $D_1 < D_2$. For comparison,

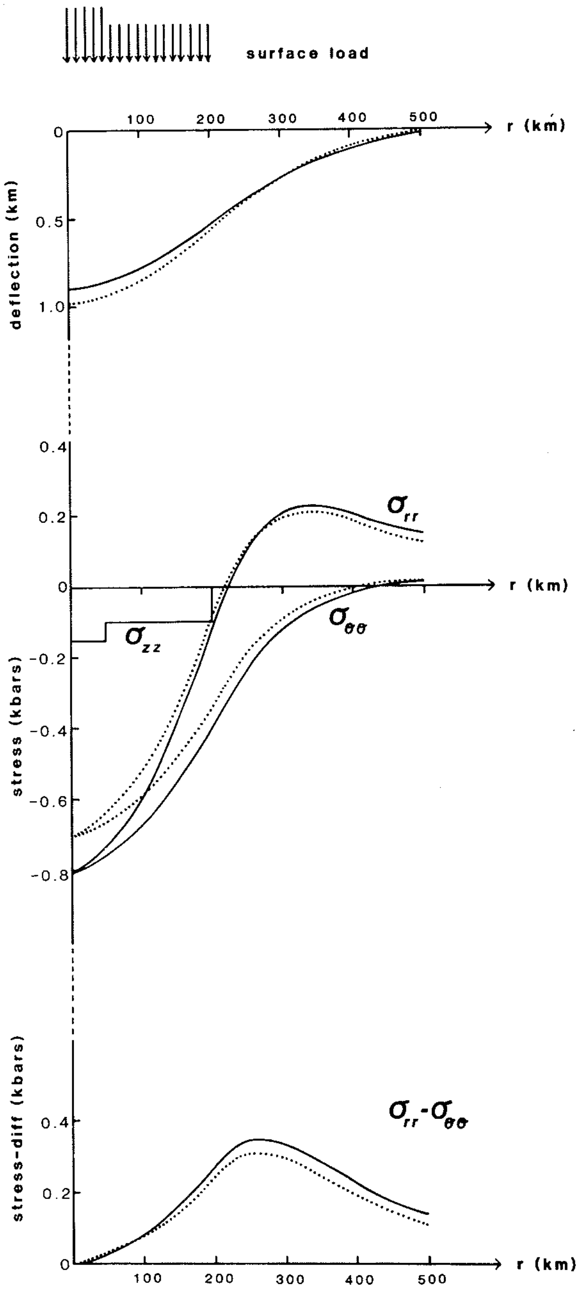

Fig. 2a. Deflection, stresses and the stress-difference $\sigma_{rr}-\sigma_{\theta\theta}$ for two laminated elastic plates as described in Fig. 2b. Solid line corresponds to model A, dotted line to model B.

Figure 5a shows the deflection and stresses for two homogeneous shells with flexural rigidities D_1 and D_2. If $r^* \ll A$ the effect of the weaker inner region is unimportant and the response is essentially that of a homogeneous shell of thickness H_2 and rigidity μ_2. As r^* increases, both regions of the shell affect the response. For the cases illustrated in Fig. 5, where $0.75\,A \leq r^* \leq 1.5\,A$, the observed deflection corresponds closely to that expected for a homogeneous shell characterized by the thickness and elastic properties of the inner

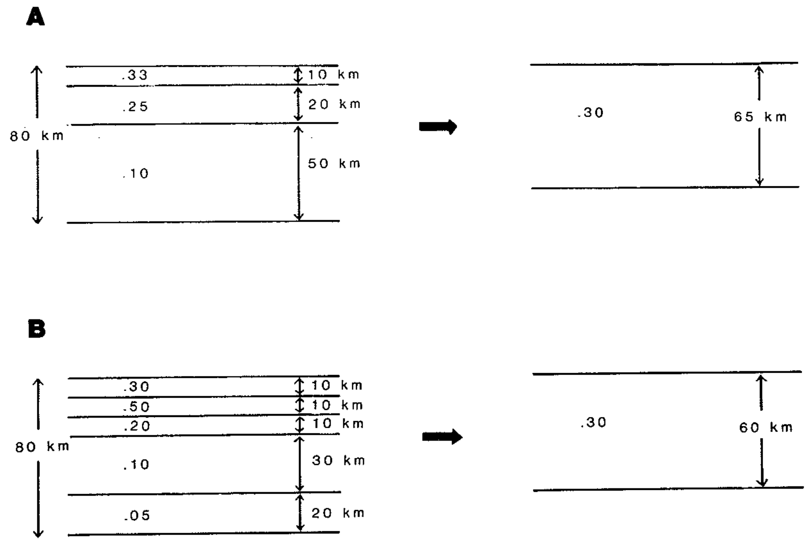

Fig. 2b. Two laminated elastic plates and their equivalent homogeneous plates. The rigidity of each layer is indicated in units of 10^{12} dynes/cm^2.

region. However, as the stresses are concentrated in the outer and stronger region, considerable variation is observed in the radial distribution of the stress components σ_{rr} and $\sigma_{\theta\theta}$ and hence in the area where one would predict failure to occur. These models with $r^* \simeq A$ may be applicable to the mascon problem if, at the time of mare flooding, the lunar lithosphere still reflected evidence of the basin-forming impact and the subsequent formation of a mantle plug. In this case further unknowns (ΔD and r^*) are introduced into the problem, and the thickness of the lithosphere estimated by the

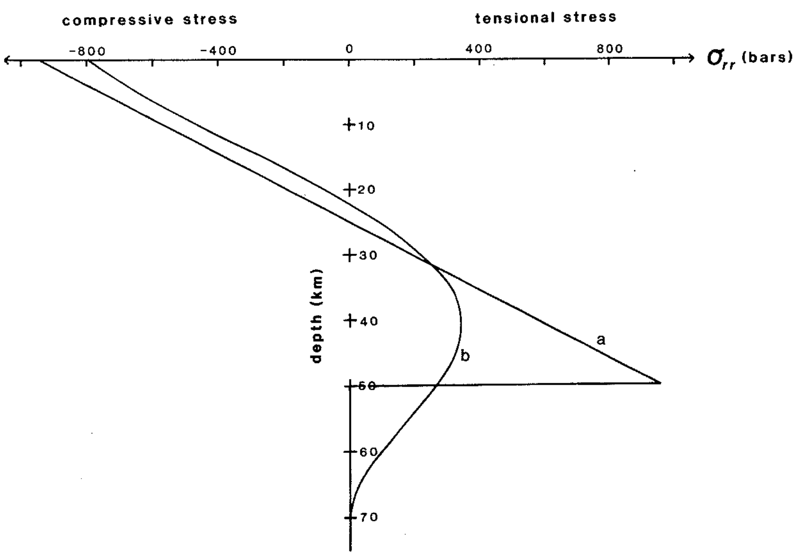

Fig. 3. Variation of stress with depth underneath the load for a 50 km thick homogeneous shell (a), and a shell with the same flexural rigidity where μ varies with depth according to the equation $\mu(z) = (\mu_0/2)[1 + \cos(\pi/H_0)(z - z_0)]$ with $z_0 = 10$ km and $H_0 = 60$ km (b).

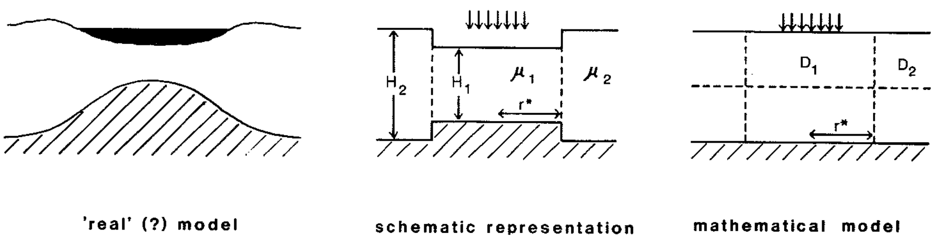

Fig. 4. Mathematical model of variable flexural rigidity used to obtain results of Fig. 5.

simple flexure theory would vary between H_1 and H_2 depending on the value of r*.

The use of simple plate theory leads to an underestimation of the plate thickness for the two reasons discussed above; namely (i) equating the elastic thickness with the thickness of the stress-bearing part of the plate leads to an underestimation of H by amounts that are rheology dependent and (ii) the introduction of a variable thickness plate may lead to a further model dependent underestimation of H. This suggests that the lithospheric thickness may be significantly greater than estimated by Solomon and Head and that the thin plate theory is therefore not a very precise indicator of deformation or of stress. While the absolute values of H given by Solomon and Head may be underestimated, the proposed variation of H with time may still be valid since the above "corrections" to the simple plate theory will, in the first instance, be similar for all mascons.

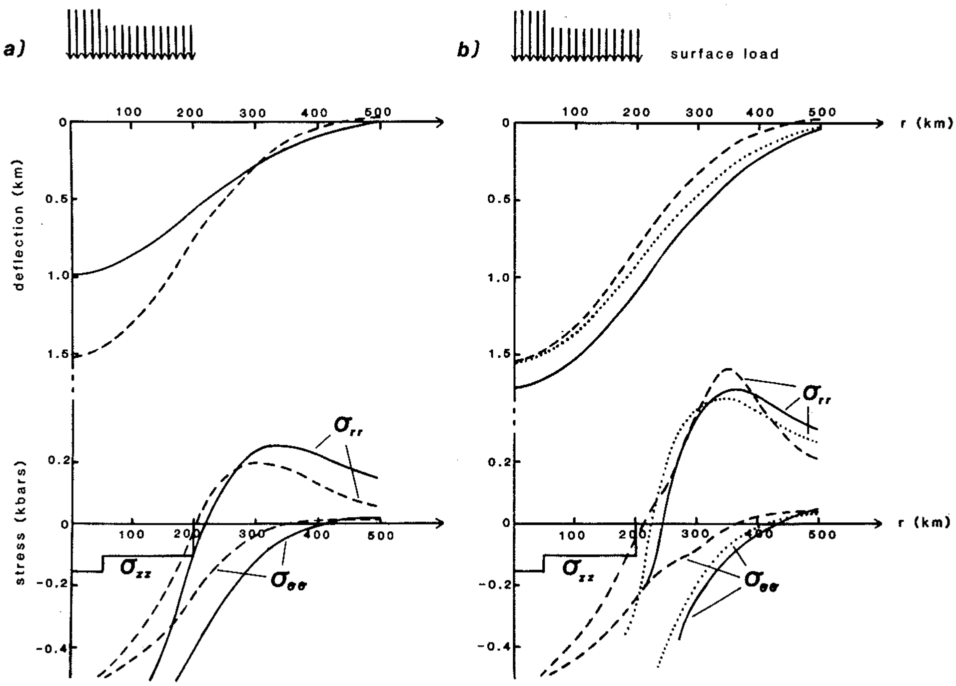

Fig. 5. a) Deflection and surface stresses for two homogeneous elastic shells characterized by flexural rigidities of $D_1 = 5.14 \times 10^{30}$ dynes-cm (solid lines) and $D_2 = 1.54 \times 10^{31}$ dynes-cm (dashed lines). These values can be interpreted in terms of homogeneous shells 60 km in thickness with rigidities of $\mu_1 = 1 \times 10^{11}$ dynes/cm^2 and $\mu_2 = 3 \times 10^{11}$ dynes/cm^2. b) A similar plot for shells characterized by a discontinuity in D: $D = D_1$ for $r < r^*$, $D = D_2$ for $r > r^*$. The dotted lines correspond to r* = 150 km, the solid lines to r* = 200 km and the dashed lines to r* = 300 km. The discontinuity in the stresses at r = r* has been smoothed over the distance r* ± 50 km.

VISCOELASTIC SOLUTIONS

An underlying assumption in the flexure models is that there is no relaxation of stress with time. This implies that temperatures T in the lithosphere cannot exceed a small fraction of the melting point temperature T_m. Usually it is assumed that creep does not occur if $T/T_M \lesssim 0.5$, but on geological time scales this fraction may be smaller. To investigate stress relaxation we have used the formalism given by Lambeck and Nakiboglu (1981) for seamounts and by Nakiboglu and Lambeck (pers. comm.) for the Lake Bonneville rebound. Models considered include viscoelastic plates, laminated elastic-viscoelastic plates, laminated elastic plates over thick viscoelastic layers and the viscoelastic half-space. In general, geophysical observations are inadequate to discriminate between the various models but their implications regarding the stress-state are quite different.

The response of a viscoelastic plate over a fluid substratum follows from the theory given by Beaumont (1978) and Lambeck and Nakiboglu (1981). At time $t = 0$ the solution corresponds to the elastic plate solution, and as $t \to \infty$ the solution approaches local isostasy. With time the deflection increases and the point where tensional σ_{rr} is a maximum moves inwards. Hence, when observations of gravity and the load are available at one epoch only, the viscoelastic solution is very similar to an equivalent elastic solution. Applying the latter theory to a viscoelastic plate results in an effective flexural rigidity which is dependent on viscosity and wavelength of the load. Like the elastic plate models the solutions are only valid for homogeneous thin plates and small deflections. In particular, when deflections become large the usual application of the correspondence principle is no longer valid (McConnell, 1965; Lambeck and Nakiboglu, 1981). Any depth dependence of the rheology or any lateral variation will have the same consequence on estimates of H as for the elastic solutions.

The viscoelastic solutions still require a rapid decrease in viscosity at a relatively shallow depth, and in the absence of convincing evidence for this, thick lithosphere models should also be explored. In particular, we have investigated models in which a thin elastic layer overlies a viscoelastic mantle. Nakiboglu and Lambeck (pers. comm.) noted that when the thickness of the viscoelastic layer is much greater than the radius of the load, the boundary conditions the base of the layer are unimportant and for this reason we consider in detail here only a viscoelastic halfspace model overlain by, and coupled to, an elastic crust.

The response of a viscoelastic halfspace to a surface load follows from the elastic halfspace solution (e.g., Sneddon, 1951) by applying the correspondence principle. This permits the Laplace transformed solution of a quasi-static, viscoelastic Maxwell body problem to be obtained from the solution of the equivalent elastic problem by replacing the elastic μ with the operator $\tilde{\mu}(s) = \mu(s/s+\tau^{-1})$. s is the Laplace transform variable and $\tau = \eta/\mu_s$ is the relaxation time for material characterized by a viscosity η and rigidity μ_s. The assumption of incompressibility considerably simplifies the problem (since $\tilde{\nu}(s) = \nu = 0.5$) without affecting the final results in any significant manner. The introduction of the elastic layer modifies the boundary condition on the upper surface by requiring this boundary to be a solution of the plate equation

$$D\,\nabla^4 w + p = q \tag{7}$$

where p is the restoring pressure applied to the halfspace. In the special case of the fluid halfspace, $p = \rho_s g w$ and Eq. (7) is equivalent to (1). For a disc surface load of density ρ_ℓ, thickness h_ℓ and radius A, applied at time $t = 0$,

$$q(r,t) = \rho_\ell g h_\ell H(A-r)\,H(t)$$

where H is the Heaviside step function. The deformation at the surface is (Nakiboglu and Lambeck, pers. comm.

$$w(r,t) = \frac{\rho_\ell}{\rho_s} h_\ell \int_0^\infty \left[\frac{1-(1-\beta)e^{-\beta t/\tau}}{1+v^4}\right] J_1(u)\,J_0(ux)\,du$$

where J_n are nth order Bessel functions, $x = r/A$, $v = \ell u/A$, $\beta = \alpha(1+v^4)/[u+\alpha(1+v^4)]$ and $\alpha = \rho_s g A/2\mu_s$. As $t \to \infty$

$$w(r) = \frac{\rho_\ell}{\rho_s} h_\ell \int_0^\infty \frac{J_1(u)\, J_0(ux)}{1+v^4}\, du$$

which is identical to (4) as can be readily demonstrated if (1) is solved by using Hankel transforms. Knowing w(r, t), the stress components follow from Eqs. (5) or (6).

At time $t = 0$ the stresses resulting from the surface load are supported by both the viscoelastic medium and by the elastic layer. With time, the stresses migrate into elastic lithosphere at a rate that is determined by the viscosity. Figure 6 illustrates the gradual

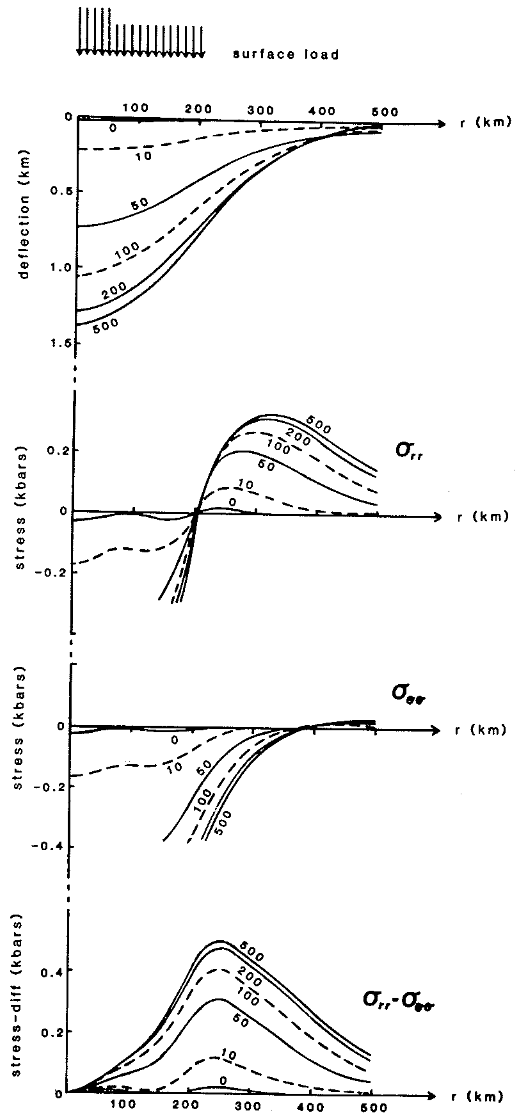

Fig. 6. Deflection, stress and the stress-difference σ_{rr} and $\sigma_{\theta\theta}$ as a function of time at the surface of a homogeneous elastic plate ($H = 50$ km, $\mu = 3 \times 10^{11}$ dynes/cm^2, $\nu = 0.30$) overlying a viscoelastic halfspace of rigidity $\mu_s = 5 \times 10^{11}$ dynes/cm^2. The curves are identified by the time since the application of the surface load in units of the relaxation time $\tau = \eta/\mu_s$.

increase in the stresses at the outer surface of a 50 km thick layer overlying a halfspace with $\mu_s = 5 \times 10^{11}$ dynes cm^{-2}. In this model the simple elastic layer solutions is reached at about $t \simeq 250\tau$. That is, if the substratum is characterized by a low viscosity, of the order 10^{23} poise, it would behave as a fluid on a time scale of $(1\text{–}2) \times 10^6$ years. This period of time is insignificant in terms of the mare flooding history, and so in this case the inviscid substratum approximation is valid. For $\eta \simeq 10^{25}$ poise, the elastic plate solution would not be reached for $(1\text{–}2) \times 10^8$ years, and during this time other evolutionary processes, such as thickening of the lithosphere due to the general cooling of the moon, may have been significant. The viscosity of the lunar mantle is essentially unknown, but published estimates have spanned at least the range given here. If the actual viscosity value is in the upper range of estimates, complete relaxation of the substratum stresses may not have been achieved before the lithosphere had thickened significantly. In this case, the lithospheric thickness estimated by the simple elastic flexure model would be an underestimation of the actual thickness of the stress-bearing layer. As well as this, Fig. 6 shows that the position of maximum radial stress at the surface moves away from the centre of the load with time (by ~100 km). If an additional extensional stress field is superimposed to explain the formation of the concentric rilles, failure will occur when the magnitude of this stress plus $\sigma_{rr}/\max$ exceeds the finite strength of the surface layer. As the tensional strength of competent rock is low, it is unlikely that the equilibrium solution (that of an elastic layer overlying a fluid) would be reached before failure occurred. Thus, once again the simple flexure theory would lead to an underestimation of the lithospheric thickness. Without the superposition of an additional stress field, the maximum stress-difference at the surface would be $\sigma_{rr} - \sigma_{\theta\theta}$. Figure 6 shows that σ_{rr} and $\sigma_{\theta\theta}$ evolve such that the position of this stress-difference does not change with time.

If the viscoelastic layer is of finite thickness the response will differ little from the halfspace model provided the thickness H_s is greater than about 3A. For H_s much less than this, both the boundary conditions at the base of the viscoelastic layer and the value of H_s become important. Appropriate boundary conditions for the moon would be those corresponding to an inviscid lower mantle. In this case the deformation and stress-state differ from the halfspace model mainly in that the stress relaxation is faster (see Nakiboglu and Lambeck, pers. comm.).

DISCUSSION

We have examined above several models in which we attempted to take into account (i) plausible departures of the lunar lithosphere from the thin homogeneous elastic shell model, and (ii) the assumption of the underlying fluid mantle. Although we have not fitted the models to real data, several observations can be made. One is that lateral variations of lithospheric properties underneath the mascon basins do have a significant effect on the distribution of surface stresses. That is, the thickness of the lithosphere estimated by using the simple flexure model to match features associated with mare basins, may be strongly ifluenced by local perturbations in the lithospheric structure. Secondly, any estimate of H must be considered as an effective elastic thickness and the actual stress-bearing layer may be considerably thicker than this. Models with depth-dependent rheologies have the particular advantage of removing the need for a major rheological discontinuity for which there is no evidence in lunar models based on seismic data. Recent revisions of the selenotherm by Huebner *et al.* (1979) also make it unlikely that an abrupt change with depth occurs in the response of the moon to near-surface loading (see also Lambeck and Pullan, 1980; Pullan and Lambeck, 1980; Delano *et al.*, 1980). For these reasons models of an elastic crust overlying a thick viscoelastic mantle have also been explored in a preliminary manner. These models also do not indicate clear-cut relations between the rheology and surface tectonics but do emphasize that the parameters resulting from elastic plate models are time-averaged parameters.

The problem of determining the thickness of the elastic lithosphere by relating the flexure stress field to the occurrence of arcuate graben, is clearly more complex and less well constrained than suggested by the simple models used by Solomon and Head (1979, 1980). It should also be stressed that in fact the flexure theory does not predict the formation of graben at the margin of the surface loads (e.g., Melosh, 1978), except in the case of very thin lithospheres, unless a regional stress field is superimposed upon the flexure-associated stress field. The magnitude of this regional stress field is not insignificant however, and probably not inconsequential in shaping other parts of the moon's surface. In fact, it should be asked whether this additional regional or global stress field invoked by Solomon and Head to explain the graben, may actually have greater significance than in merely modifying the stress field induced by flexure. For example, what are the consequences of the tensional stresses of a few hundred bars required to annul the compressional flexure-associated $\sigma_{\theta\theta}$? Also, what is the evidence that "loading of mare basins and the consequent vertical subsidence as the lithosphere adjusted to the load are the *primary* causes of basin related graben" (Solomon and Head, 1980, p. 131)? Could pre-existing stress-distributions and zones of weakness associated with the impact basin structures not be sufficient to modify the global stress field so as to cause the lunar rilles?

Acknowledgments—We thank Sean Solomon and James Head for their strenuous opposition to an earlier version of this paper. It led us to a better understanding of their 1979 paper and avoided a possibly embarrassing situation.

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