JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 90, NO. B4, PAGES 2985–2994, MARCH 10, 1985

Thermal Response of a Moving Lithosphere Over a Mantle Heat Source

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The mid-ocean bathymetric swells observed around some volcanic island chains can be modeled using the thermal conduction equations of an oceanic lithosphere moving over a heat source located at the base of the plate. Some previous studies of this problem have suggested that the heat source must lie well within the lithosphere in order to explain the observed bathymetry and surface heat flow. These conclusions are based on an isothermal boundary condition at the base of the lithosphere, but this choice requires that the heat source is maintained above this boundary and can actually lead to a flux of heat back into the mantle, particularly in the vicinity of the heat source. If the lower boundary is insulated insofar as the anomalous temperature field is concerned, flux from the heat source back into the mantle is prevented, resulting in a more rapid uplift of, and in an increased heat flow across, the seafloor. Previous solutions have considered only the steady state solutions, whereas the present model also evaluates transient solutions. For rapidly moving oceanic plates the transient effects can be neglected provided that the heat source has been active for at least 10^7 years and that the direction of motion and the intensity of the heat source have remained constant in this interval. Solutions, with the heat source located near the base of the conductive layer, of the conduction equations for heat flow, bathymetry, and geoid heights in the spatial domain reproduce the principal observational characteristics of the Hawaiian swell at distances away from the immediate origin of the swell, and this part of the swell can be used to estimate the thermal conduction model parameters. The model does not wholly explain the observations near the origin and the mismatch may reflect a dynamic component in the support of the swell.

INTRODUCTION

Within the framework of the plate tectonics hypothesis the thermal evolution of the lithosphere is adequately explained by models of conductive heat transfer in which newly formed lithosphere at ocean ridges cools and contracts as it ages and spreads away from the ridge axis [e.g., *McKenzie*, 1967; *Turcott and Oxburgh*, 1967]. Much of the observed ocean bathymetry and heat flow can be explained by these models. Specifically, the bathymetry is proportional to the square root of the age of the lithosphere as is the heat flow [e.g., *Sclater and Francheteau*, 1970; *Anderson et al.*, 1973; *Parsons and Sclater*, 1977; *Heestand and Crough*, 1981], but exceptions do occur. Most notable are the swells surrounding some volcanic island chains such as Hawaii, the Cook-Australs, Bermuda, and Cape Verde. Here, the regional bathymetry may be as much as 1000 m above the surrounding seafloor [*Dietz and Menard*, 1953; *Menard*, 1973] and above that predicted by the lithosphere cooling models. It has been suggested that these swells, and possibly the associated midplate volcanism, are the result of anomalously high temperatures, or hotspots, within or below the lithosphere [*Wilson*, 1963; *Morgan*, 1972; *Detrick and Crough*, 1978]. This appears to be confirmed by the observations of heat flow, with the surface heat flow along the Hawaiian swell being reported to be about 20–25% higher than for "normal" ocean lithosphere of the same age as predicted by the conductive cooling model [*Detrick et al.*, 1981; *Von Herzen et al.*, 1982]. These authors also note that the anomalous heat flow increases from the present center of volcanic activity at Hawaii, where heat flow is normal, to values that are 20–25% above normal at Midway. The majority of the midplates swells rise to a depth that is comparable to depths of 25 Ma old lithosphere and then subside at a rate that is in accordance with the conductive cooling of the rejuvenated plate [*Crough*, 1978]. This is usually taken as evidence that the conductive heating and cooling is the dominant mechanism of heat transfer and in controlling the regional bathymetry. However, magmatism and convective heat transport may be important within the "hotspot" area [e.g., *Withjack*, 1979].

In order to explain the steep upwelling at the upstream end of the swell it has been argued that the conductive reheating model requires very rapid heat advection and a rather shallow heat source situated in the upper half of the lithosphere [*Sandwell*, 1982; *Detrick and Crough*, 1978]. However, our numerical experiments indicate that such extreme thinning leads to much faster subsidence and to higher heat flow anomalies than is observed. Instead, the difficulty with the uplift at the upstream end of the swell can be partly resolved by introducing a more realistic thermal boundary condition at the base of the lithosphere (see below).

An alternative suggestion for the origin of the ocean swells is that they are a consequence of dynamic forces acting on the base of the plate and associated with mantle convection [e.g., *Watts*, 1976; *McKenzie*, 1977; *McKenzie et al.*, 1980]. The overall observational characteristics of such a model are similar to those of the static conduction model since in both instances uplift is proportional to $3\alpha v$, α being the linear thermal expansion coefficient and v the anomalous temperature of the mantle in the dynamic model and of the lithosphere in the static model. Both processes probably combine in producing the observed heat flow and bathymetry that characterize the major ocean swells, with the dynamic process probably being more important at the active end of the swell and the static process governing the subsidence farther away.

A formulation for conductive heating of a lithosphere moving relative to a point source was given by *Birch* [1975]. This was the steady state solution of *Carslaw and Jaeger* [1959] for a half space with its surface kept at a constant temperature. This model tacitly assumes that the hotspot conductively heats up the sublithosphere mantle as well as the lithosphere and if used to interpret surface observations, it will

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Published in 1985 by the American Geophysical Union.

Paper number 4B5274.

lead to an overestimation of the heat source. More recently, *Mareschal* [1981] derived the Green's functions for surface heat flow and uplift of a stationary lithosphere with an anomalous heat flux at its base. This model is not directly applicable to the oceanic plates which are presumed to move rather rapidly relative to the hotspots. *Sandwell* [1982] considered the problem of the lithosphere moving with respect to a stationary heat source that lies within the lower part of the layer such that the plate is effectively pulled through the heat source. His solutions are for the steady state condition and are given as Fourier transforms of the Green's functions for temperature, surface uplift, and geoid perturbation. Sandwell adopts an isothermal boundary condition at the base of the plate, but this choice can actually lead to a flux of heat back into the mantle, particularly in the vicinity of the source, and it is this that dictates that the source lies within the lithosphere if this model is to represent the rapid uplift rate at the younger end of the Hawaiian swell [*Detrick and Crough*, 1978]. In fact, the isothermal boundary condition in the neighborhood of the hotspot requires that the heat source is maintained above the boundary surface, not below in the region that does not participate in the plate motion: The adoption of the isothermal boundary has the immediate consequence that the heat source must be above it and within the moving plate. A more appropriate lower boundary condition may be one of zero heat flux, one that is insulated insofar as the anomalous temperature field is concerned. This condition prevents flux of the anomalous heat back into the mantle and, all other parameters being equal, results in a more rapid uplift of, and in an increased heat flux across, the seafloor. This boundary condition represents the other extreme to the isothermal model but may be more representative of the actual conditions at the base of the lithosphere. A more rigorous formulation should incorporate the earth's mantle as the second layer in which the "hotspot" is maintained as a three-dimensional vertical plume from great depth.

In this paper we consider both the transient and steady state solutions of the thermal conduction into a plate moving over a stationary source, beneath or within the layer, under a general set of boundary conditions. The resulting solutions permit not only an examination of the plate in the not insignificant time interval required before the steady state is reached but also the treatment of time dependent heat sources or of time dependent plate motions. We make some qualitative comparisons of the predicted heat flow, bathymetry, and geoid fields with observations of the Hawaiian swell. A principal conclusion drawn is that much of the older part of the swell, greater than about 5 Ma, can be attributed to the conductive heat transfer with a source located beneath the lithosphere. The model does not adequately represent the observed fields for younger ages, and this discrepancy may be indicative of a dynamic contribution to the support of this part of the swell.

GENERALIZED GREEN'S FUNCTIONS FOR TEMPERATURE AND SURFACE HEAT FLOW

Consider a lithospheric plate moving over or through a heat source. The resulting anomalous temperature field v is a consequence of the conductive heating of the plate by the source and is superimposed upon an ambient steady state field v_0. The plate is taken to be a homogeneous isotropic solid with thermal properties that are independent of temperature. No changes of state are assumed to occur in response to the temperature perturbation. The governing equation for a stationary medium is

$$\nabla^2 v + \frac{Q}{K} = \frac{1}{\kappa}\,\partial_t v \tag{1}$$

where K is the thermal conductivity and κ is the thermal diffusivity of the plate. Q is the rate at which heat is produced in the solid, per unit time per unit volume, and will be a function of both position and time. ∇^2 is the Laplacian operator. In cylindrical polar coordinates (r, θ, z) appropriate for problems with cylindrical symmetry,

$$\nabla^2 = \partial_{rr} + r^{-1}\partial_r + \partial_{zz}$$

Also $\partial_t = \partial/\partial_t$, and $\partial_{rr} = \partial^2/\partial r^2$, etc. We take the horizontal x-y plane to coincide with the lithosphere-asthenosphere interface, and the z axis is directed upward.

The initial condition for a heat source applied at time $t = 0$ is

$$\lim_{t\to 0^-} v(r, z, t) = 0 \tag{2}$$

The boundary conditions for the anomalous temperature field can be written in a general way as [cf. *Carslaw and Jaeger*, 1959, pp. 18–25]

$$(-k_1\partial_z v + h_1 v)|_{z=0} = 0 \tag{3a}$$

$$(k_2\partial_z v + h_2 v)|_{z=d} = 0 \tag{3b}$$

where d is the plate thickness. The k_i and h_i are constants and the Kh_i/k_i are sometimes referred to as the surface conductance. Three special cases can be deduced from (3): (1) isothermal boundary ($k_i = 0$), (2) insulated or zero heat flux boundary ($h_i = 0$), and (3) convection boundary condition according to Newton's law of cooling. In the present formulation we consider sources at arbitrary locations within or on the boundary of the plate with the case of a hotspot heating the plate from below being treated by placing the source Q on the boundary.

The heat source is initially assumed to be stationary relative to the lithosphere. Its intensity is described by Dirac delta functions of position and time as

$$Q(r, z, t) = \delta(r - r')\delta(z - z')\delta(t - t') \tag{4}$$

where r', and z' are the coordinates of the impulse point source and t' is the epoch at which this source is applied. With the assumption of cylindrical symmetry and homogeneous initial conditions, the boundary value problem defined by (1)–(4) can be solved using Hankel and Laplace transform techniques. The point source solution for the anomalous temperature field is given in Appendix 1 (equation (A7*a*)) and the solution for a moving source (or moving lithosphere), distributed over a finite area and applied for a finite period of time, follows from the convolution of this point source solution with appropriate spatial and temporal distribution functions. The Green's functions for the temperature field, for a lithosphere in uniform motion relative to a point source located within the plate or at its base, are given by equations (A8) and (A9).

The anomalous surface heat flow is

$$\Omega = -K\mathbf{n}\cdot \operatorname{grad} v$$

or

$$\Omega = -K\partial_z v|_{z=d} \tag{5}$$

where $\mathbf{n}$ is the outward unit normal vector of the surface across which the flux is measured. From equations (A7), (A8), and (A9) the Green's function for surface heat flow due to a

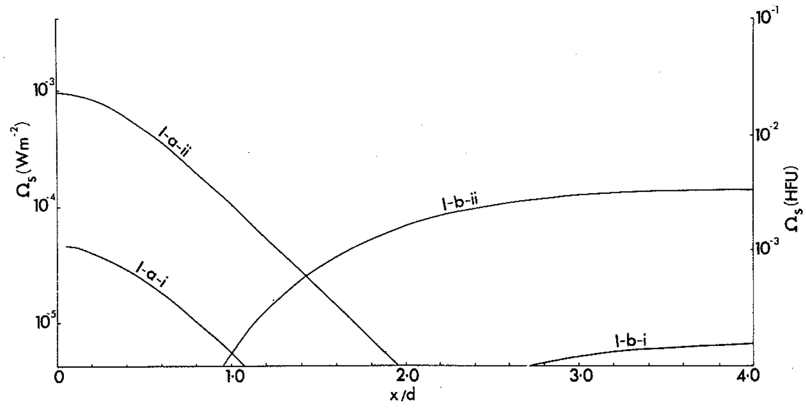

Fig. 1*a*

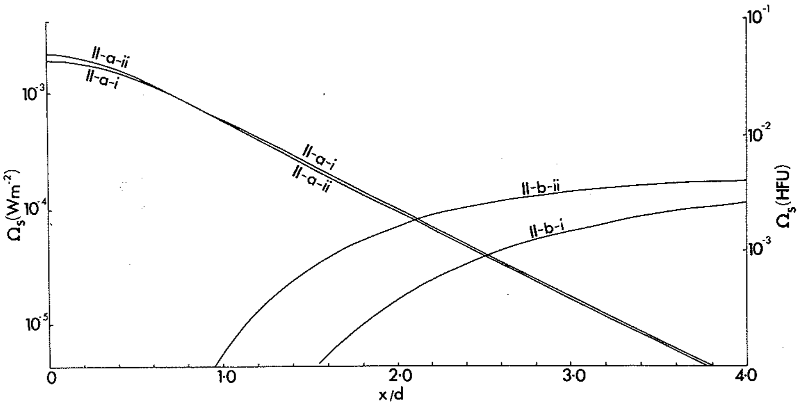

Fig. 1*b*

Fig. 1. Steady state surface heat flow for an 80-km-thick lithosphere heated by a point source of intensity $Q = 4.19 \times 10^6$ W (10^6 cal s^{-1}) located at $x' = 0$. The following notation for the various results is adopted. I, isothermal boundary condition at the base of the plate; II, insulated boundary condition; *a*, zero plate velocity; *b*, $u = 0.02$ m yr^{-1}; i, heat source located on the lower boundary ($z' = 0.01d$); ii, heat source located within the plate at $z' = 0.2d$.

source located at $x' = y' = 0$ and z' is

$$G_\Omega(x, y, t) = \frac{Q}{2\pi d^2} \sum_{n=1}^{\infty} B_n Z_n'(z) Z_n(z') I_n(x, y, t) \qquad (6)$$

with $Z_n'(z) = d^2 v_n(-k_1 v_n \sin v_n d + h_1 \cos v_n d)$ and the eigenvalues v_n defined by equation (A7*b*).

BOUNDARY CONDITIONS

The Green's functions developed above are for the general boundary conditions given in (3). If the seafloor is covered by a thick layer of compacted sediments, the heat escape through the upper boundary surface ($z = d$) is proportional to the ratio (sediment conductivity/sediment thickness) [*Carslaw and Jaeger*, 1959, pp. 20–21] and is generally inefficient. In the absence of sediments the overlying water body transfers heat away from the plate predominantly by natural convection and now the heat flux across the ocean floor at $z = d$ is proportional to the 5/4 power of the temperature difference between the body and the water [*Carslaw and Jaeger*, 1959, p. 21]. For thin or unconsolidated sediments a heat flux proportional to temperature appears more appropriate. Heat flow observations across undisturbed ocean lithosphere indicate that the quantity $(h_2 d/k_2)$ is of the order of $10^2 \sim 10^3$ and such a large value permits of a simplified boundary condition at the ocean floor, irrespective of the nature of the boundary condition at the base of the lithosphere. First, taking the lower boundary to be insulated, then

$$\partial_z v|_{z=0} = 0$$
$$(k_2 \partial_z v + h_2 v)|_{z=d} = 0 \qquad (7)$$

With the conditions (7) the eigenvalues of the governing differential equation follow from equation (A7*b*) as

$$v_n d \tan v_n d = hd \qquad (8a)$$

where $h = h_2/k_2$. The smallest positive root of (8*a*) is $vd = 1.5552$ for $hd = 100$, and it is $\pi/2$ for $hd \rightarrow \infty (k_2 \rightarrow 0)$. The difference is only about 1% and a reasonable approximation of the upper surface boundary condition is obtained by setting $k_2 = 0$. Second, taking the lower boundary to be isothermal, then

$$v_n d = -hd \tan v_n d \qquad (8b)$$

The smallest positive root of this equation is $v_n d = 3.1105$ for $hd = 100$, and π for $hd \rightarrow \infty$. The difference is again negligible, and the condition $k_2 = 0$ is adequate for this case as well. Hence the ocean floor can be taken as an isotherm, and

$$(-k_1 \partial_z v + h_1 v)|_{z=0} = 0$$
$$v|_{z=d} = 0 \qquad (9a)$$

The base of the undisturbed lithosphere can be considered

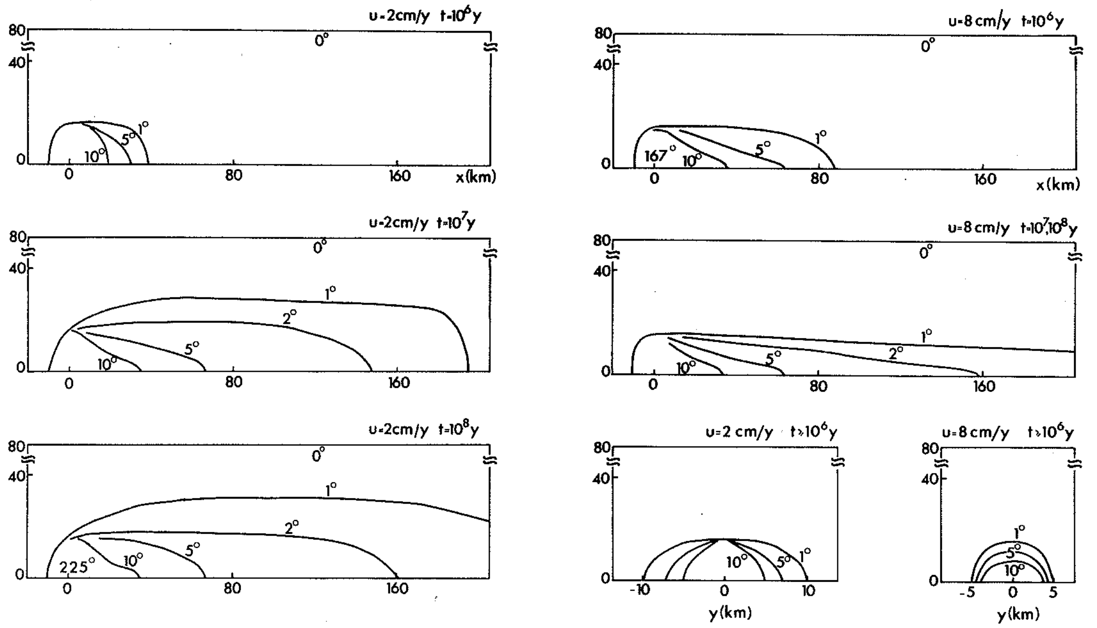

Fig. 2. Transient Green's functions for isotherms in the ocean lithosphere in the $y = 0$ plane (along the direction of plate motion), and (lower right) in the $x = 0$ plane (orthogonal to plate motion). The point source is located at $x, y, z = 0, 0, 0$, has an intensity of 4.16×10^6 W (10^6 cal s^{-1}). The contours are in degrees Celsius. The steady state in the y direction is reached very quickly, and the results on the right are independent of time for $t \geq 10^6$ years.

as an isotherm, but in the presence of an anomalous heat source at the base the isotherms are perturbed as is the lower boundary of the lithosphere. It is unlikely that the heated lithosphere releases some of its heat back into the asthenosphere from where the anomalous heat source presumably originated in the first place, and a more plausible approximation is a boundary that does not permit this return flux or one that is insulated with respect to the anomalous temperature. Thus the simplified boundary condition appropriate for the ocean lithosphere around the heat source is

$$\partial_z v|_{z=0} = 0 \qquad v|_{z=d} = 0 \tag{9b}$$

The resulting positive eigenvalues, from equation (A7*b*), are

$$\nu_n = (n - \tfrac{1}{2})\frac{\pi}{d} \qquad n = 1, 2, \cdots \tag{10}$$

The Green's functions for temperature and surface heat flow follow by substituting (10) into the relevant expressions given in Appendix 1. The results are

$$G_v(x, y, z, t) = \frac{Q}{2\pi K d}\sum_{n=1}^{\infty} \cos\left[(n - \tfrac{1}{2})\pi\frac{z}{d}\right] \cdot \cos\left[(n - \tfrac{1}{2})\pi\frac{z'}{d}\right] I_n(x, y, t) \tag{11a}$$

and

$$G_\Omega(x, y, z, t) = \frac{Q}{2d^2}\sum_{n=1}^{\infty}(-1)^n(n - \tfrac{1}{2}) \cdot \cos\left[(n - \tfrac{1}{2})\pi\frac{z'}{d}\right] I_n(x, y, t) \tag{11b}$$

with

$$I_n(x, y, t) = \exp\left(\frac{ux}{2\kappa}\right)\int_{\zeta=0}^{t}\frac{d\zeta}{\zeta} \cdot \exp\left\{-\frac{r^2}{4\kappa\zeta} - \left[\frac{u^2}{4\kappa} + (n - \tfrac{1}{2})^2\frac{\pi^2}{d^2}\kappa\right]\zeta\right\} \tag{12a}$$

Comparable results for the isothermal lower boundary follow by setting $\nu_n = n\pi/d$ ($n = 1, 2, \cdots$) into the equations given in Appendix 1.

The consequence of the choice of lower boundary condition on the surface observables can be significant. This is illustrated in Figure 1 for a nominal heat source. The steady state Green function for the surface heat flow G_Ω has been computed here for the two different boundary conditions at $z = 0$; namely the isothermal condition $v = 0$ (curves I, Figure 1*a*) and the insulated boundary $\partial_z v = 0$ (curves II, Figure 1*b*). The former condition underestimates the surface heat flow when compared with the insulated boundary condition, and the dif-

TABLE 1. Definitions and Values of Parameters Used in Model Calculations

Parameter	Definition	Value	Source
K	thermal conductivity	2.1 W m^{-1} deg^{-1}	*Clark* [1966]
α	coefficient of linear thermal expansion	5×10^{-6} deg^{-1}	*Skinner* [1966]
c	specific heat	840 J kg^{-1} deg^{-1}	*Carslaw and Jaeger* [1959]
ρ	mean lithospheric density	3.3×10^3 kg m^{-3}	
κ	thermal diffusivity	7.6×10^{-7} m^2 s^{-1}	

ference may be more than an order of magnitude if the heat source lies near the base of the plate or if the plate velocity is small (compare, for example, curves I-a-i with II-a-i). This difference reflects the flow of heat back into the mantle through the lower boundary when the isothermal boundary condition is imposed, and for deep heat sources this flux is much greater than the upper surface flux. The extent of the region of anomalous heat flow is similarly influenced by the choice of the boundary condition at the base of the lithosphere with the isothermal condition yielding a smaller region of anomalous heat flow; by a factor of 2 in radius if the plate velocity is small.

TRANSIENT EFFECTS OF ANOMALOUS HEAT SOURCE

In earlier studies [*Mareschal*, 1981; *Sandwell*, 1982] it was assumed that the heat source had been active for a sufficiently long period of time for the transient temperature field to be negligible. However, the dimensionless time, or the Fourier number, $\kappa t/d^2$, appropriate for the oceanic lithosphere is such that the transient effects are reduced by e^{-1} of the original value in (5–10) $\times 10^7$ years, a time scale that is comparable to the age of the ocean floor beneath, for example, the Hawaiian islands. Transient effects may therefore be significant in discussing mid-ocean swells. The behavior of the temperature Green's function for transient conduction is illustrated in Figure 2. In these calculations the point source of $Q = 4.2 \times 10^6$ J s^{-1} (10^6 cal s^{-1}) is located on the lower boundary at $x, y = 0$ of a layer with $d = 80$ km. The x and y axes coincide with the direction of the plate motion and the transverse direction, respectively. Other thermal parameters are summarized in Table 1. The temperature distribution in the y direction, normal to plate motion, attains the steady state quickly, whereas it takes much longer to reach this state in the x direction parallel to the plate motion. Here the required time is approximately inversely proportional to the plate velocity. The singularity of the Green's function that occurs at the point source is removed upon the convolusion of this function over the volume of a distributed heat source.

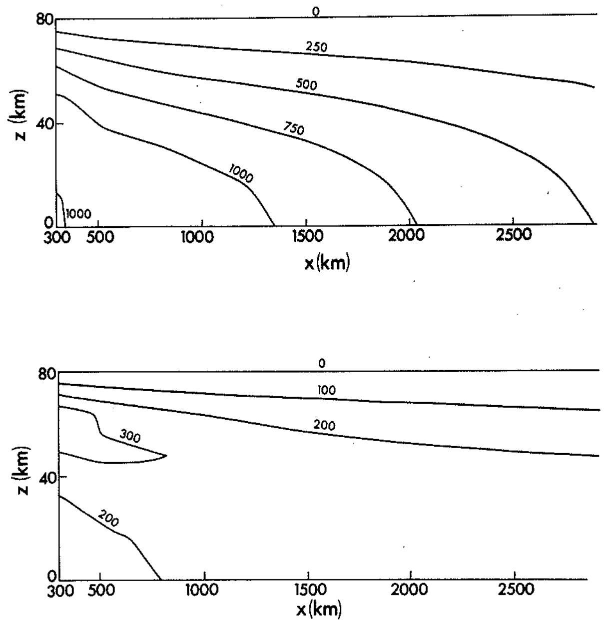

Fig. 3. The steady state isotherms for the anomalous temperature field in the ocean lithosphere downstream from the hotspot in a plane along the plate motion. The heat source of radius $R = 200$ km and intensity $q = 25$ HFU (1.05 W m^{-2}) is located at the base of the lithosphere. The contours are in degrees Celsius. The plate velocity is $u = 0.02$ m yr^{-1} (top) and $u = 0.08$ m yr^{-1} (bottom).

The anomalous temperature field of the source will also be overestimated because some heat goes into the partial melting of the lithosphere. An approximate way of taking this into account is to replace the specific heat by an effective value [*Carslaw and Jaeger*, 1959, p. 289]

$$c_{\text{eff}} = c + \frac{L}{\Delta v_m}$$

where L is the latent heat of fusion and Δv_m is the melting temperature range of the lithospheric material. A still simpler treatment is to reduce the theoretical temperatures in the region of partial melting by L/c. If the lithosphere is defined by an isotherm representing the onset of partial melting, thinning of the plate may first occur around the source and then spreads with time in the direction of plate motion with the amount of thinning being approximately inversely proportional to the plate velocity.

The calculations illustrated in Figure 2 show that the transient effects of the hotspot can be neglected for the ocean lithosphere if the hotspot has been active for more than 10^7 years, provided that the direction and velocity of the plate motion remained constant during this period. The Green's functions for the steady state temperature conditions follow by taking the limit $t \to \infty$ in (12*a*), or

$$I(x, y, \infty) = \lim_{t\to\infty} I(x, y, t)$$

$$= 2 \exp\left(\frac{ux}{2\kappa}\right) K_0 \left\{ r\left[\frac{u^2}{4\kappa^2} + (n - \tfrac{1}{2})^2 \frac{\pi^2}{d^2}\right]^{1/2} \right\} \qquad (12b)$$

where K_0 is the modified Bessel function. The appropriate steady state Green's functions follow upon substituting (12*b*) into (11*a*) and (11*b*).

SOLUTIONS FOR HEAT SOURCES OF FINITE AREAL EXTENT

Denoting the steady state Green's function for the anomalous temperature field by $G_v^\infty(x, y, z)$, the convolution over a heat source with volume τ and intensity $Q(x, y, z)$ gives the steady state temperature field:

$$v(x, y, z) = \int_\tau G_v^\infty(x - x', y - y', z - z') \cdot Q(x', y', z')\, dx'\, dy'\, dz' \qquad (13)$$

where x, y, z are the coordinates of the lithospheric point at which v is evaluated and x', y', z' specify the position of the hotspot points. The coördinate system is attached to the heat source, and the lithospheric point coordinates will be functions of time due to the plate motion. Analytical solutions can be found for some simple distributions of heat source intensity including uniform and some exponential distributions (see Appendix 2), but the numerical evaluation of these solutions involve summations over the orders of modified Bessel functions and it is equally practical to evaluate the result directly from (13) using quadrature techniques.

The anomalous temperature field is illustrated in Figure 3 for a circular heat source, of radius 200 km, located at the base of the plate and centered at $x = 0$, $y = 0$. A uniform intensity of $Q = 1.05$ W m^{-2} (25 μcal cm^{-2} s^{-1}) is adopted. These parameters are consistent with an observed surface swell of about 1 km amplitude that is locally compensated throughout the entire lithosphere. The hotspot area has not been included in the figure because any partial melting has not been modeled so that the computed temperatures in the vicinity of the source are overestimated.

The heat flow corresponding to the same heat source is illustrated in Figure 4. Surface heat flow decreases, and the location of the maximum heat flow moves downstream as the plate velocity increases, downstream shift being approximately proportional to the plate velocity and thickness. Also, the thinner the plate, the greater the surface heat flow. Preliminary calculations using heat sources of uniform, parabolic, and Gaussian areal distributions indicate that the surface heat flow is not sensitive to such fine details of heat distribution.

APPLICATION TO HAWAII

The classical mid-ocean swell is that of Hawaii [*Dietz and Menard*, 1953], and the geophysical observations of it have been discussed by *Crough* [1978], *Von Herzen et al.* [1982], and *Sandwell and Poehls* [1980]. Only a qualitative comparison between theory and observations is attempted here with the emphasis being on highlighting the importance of the choice of lower boundary condition. The model results are illustrated in Figures 5, 6, and 7. A circular heat source with Gaussian distribution has been adopted,

$$Q(r) = Q_0 \exp\left[-(r/R)^2\right]$$

where R is the radius of the heat source. The velocity of the Pacific plate relative to a fixed mantle hotspot is taken to be ~ 9.0 cm yr^{-1} [*Minster and Jordan*, 1978].

In the ensuing calculations the thermal anomalies in the lithosphere are assumed to be compensated locally, yielding the surface isostatic uplift [*Morgan*, 1975]

$$\Delta h(x, y) = \frac{3\rho_0 \alpha}{\rho_0 - \rho_w} \int_{z=0}^{d} v(x, y, z)\, dz \qquad (14)$$

where ρ_0 and ρ_w are the densities of unperturbed lithosphere and water, respectively, and α is the coefficient of linear thermal expansion. The assumption of local isostasy is justified if, as in the case of Hawaiian swell, the extent of bathymetric anomalies and temperature perturbances exceed ~ 150 km [*Nakiboglu and Lambeck*, 1985]. The corresponding isostatic geoid anomalies follow from

$$N(x, y) = \frac{G}{g} \int_{\tau} \frac{\Delta\rho}{l}\, d\tau$$

where $\Delta\rho$ is the density perturbations including those due to replacement of water with crust and those arising from the uplift of the base of the lithosphere. G and g are the gravitational constant and gravity acceleration, and l denotes the distance between geoid point and volume element $d\tau$. Assuming that the density contrast at the base of the lithosphere is zero,

$$N(x, y) = \frac{G}{g}(\rho_0 - \rho_w) \int_{x'} \int_{y'} \ln \left\{ \frac{h + [(x - x')^2 + (y - y')^2 + h^2]^{1/2}}{h - \Delta h(x', y') + [(x - x')^2 + (y - y')^2 + (h - \Delta h(x', y'))^2]^{1/2}} \right\} dx'\, dy' - \frac{G}{g} 3\alpha\rho_0 \int_{x'} \int_{y'} \int_{z'=h}^{d+h} \frac{v(x', y', z')\, dx'\, dy'\, dz'}{[(x - x')^2 + (y - y')^2 + z'^2]^{1/2}} \qquad (15)$$

where h is the depth of the water overlying unperturbed lithosphere. An approximate expression of geoid height for a lithosphere in Pratt isostasy is given by *Haxby and Turcotte*

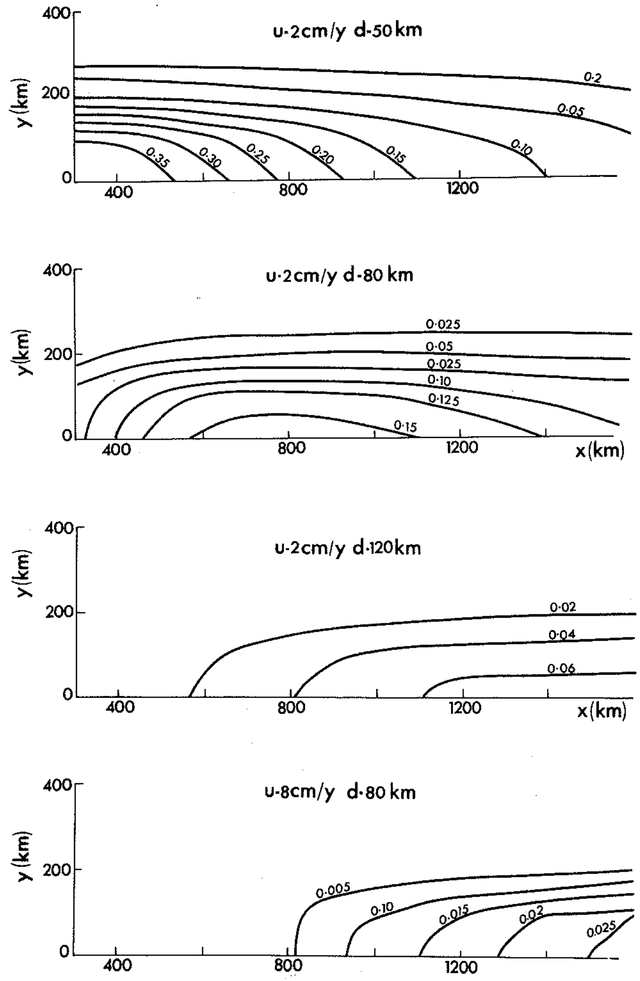

Fig. 4. The steady state anomalous surface heat flow in the ocean lithosphere downstream from the heat source. The heat source characteristics are as in Figure 3. The contours are in watts per square meter.

[1978] as

$$N(x, y) \simeq \frac{\pi G}{g} D(\rho_0 - \rho_w)\, \Delta h(x, y) \qquad (16)$$

where D is the depth of compensation which is here taken to be at the base of the lithosphere. We have compared the two equations for various values of lithospheric thickness and hotspot radius by integrating numerically the rigorous expression (15). The results show that Haxby-Turcotte approximation underestimates the geoid height by an amount inversely proportional to hotspot radius, but the error is nearly constant everywhere and it never exceeds 20% of the maximum geoid height. For a hotspot radius in excess of 200 km the error is below 10%.

Figures 5*a* and 5*b* illustrate the observed and calculated bathymetric profiles across the swell at 204° longitude and along the swell, respectively. The "observed" longitudinal profile was obtained by correcting the 1° world height data for

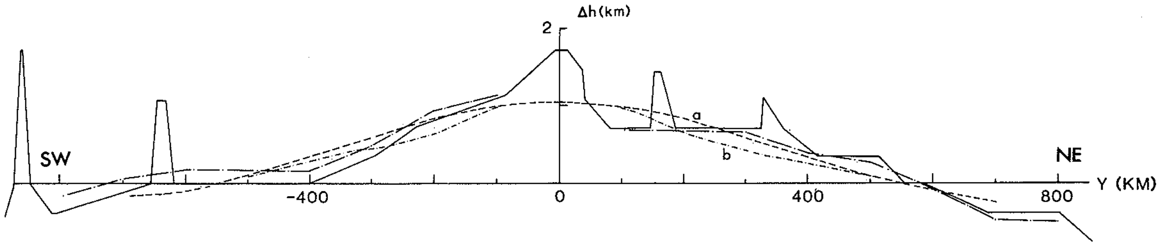

Fig. 5*a*

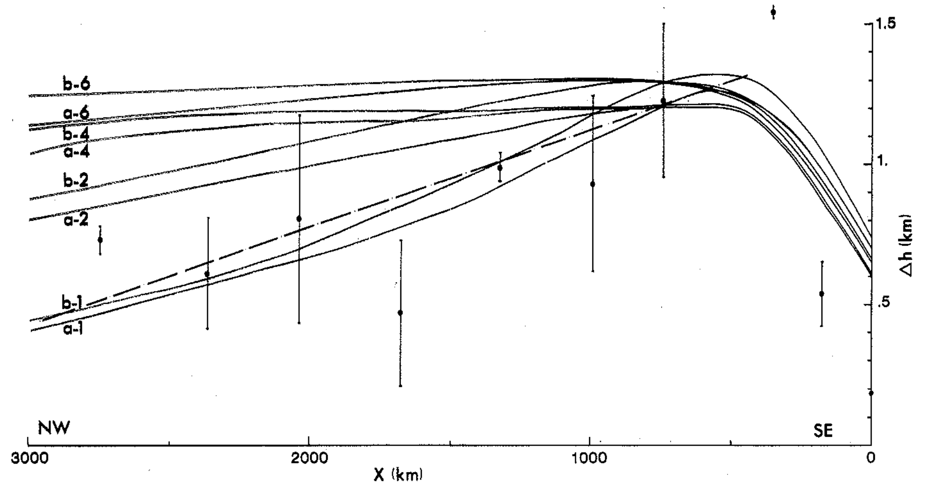

Fig. 5*b*

Fig. 5. (*a*) Observed bathymetric profile in a direction perpendicular to the Hawaiian swell taken at 204° longitude (solid line); the long-wavelength bathymetry after seafloor age correction (dash-dot line); the calculated profiles (dashed lines) for curve a, $R = 600$ km, and curve b, 700 km. (*b*) The long-wavelength bathymetry along the center of the Hawaiian swell corrected for age (solid circles) together with the uncertainties associated with filtering and age corrections (error bars). The dash-dot line is the linear regression line best fitting the observation points excluding the single upstream point. The solid lines are the theoretical profiles for hot spot radius: curve a, $R = 600$ km, and curve b, 700 km. The lithospheric thickness is 1, 40 km; 2, 60 km; 3, 70 km; 4, 80 km; 5, 90 km; and 6, 100 km. The intensity of the heat source required to give the profiles above is inversely proportional to plate thickness and range from 0.48 W m^{-2} for $d = 100$ km to 0.59 W m^{-1} for $d = 40$ km.

age of the lithosphere and then averaging these values over 2° blocks. Figure 6 gives the calculated isostatic geoid together with the observed values (solid dots) which were obtained by averaging GEOS 3 altimeter data over 2° blocks and filtering out the 12° geoid as deduced from global satellite models. The predicted heat flow is illustrated in Figure 7 for different values of source radius R and plate thickness d. The heat source intensity has been estimated by fitting the model to the transverse profile. The calculations in the x direction are along a line situated 250 km from the swell axis so that it can be compared with the observations of *Detrick et al.* [1981] south of the swell axis. These heat flow measurements have been corrected by Detrick et al. for the decay with lithospheric age and the residual values range from 3.3 to 13 mW m^{-2}. These magnitudes suggest that the thermal thickness of the unperturbed lithosphere is from 70 to 90 km.

What is significant in Figures 5–7 is not so much the degree of match or mismatch between theory and observations but that a source at the base of the "thermal lithosphere" can produce the correct order of magnitude results at distances

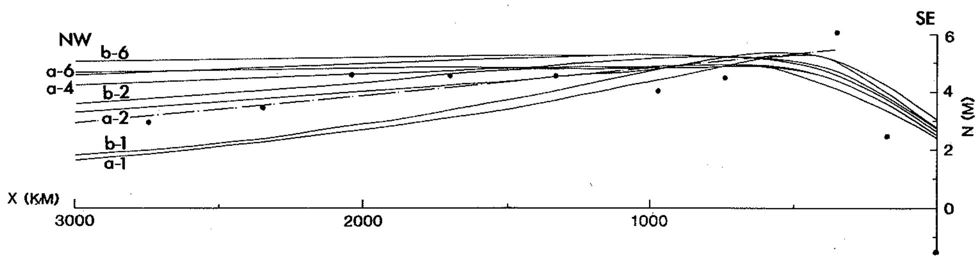

Fig. 6. The long-wavelength geoid (solid circles) and theoretical isostatic values (solid lines) along the center of the Hawaiian swell. The dash-dot line is the linear regression line obtained from the observation points. The hotspot radius and lithosphere thickness are as in Figure 5.

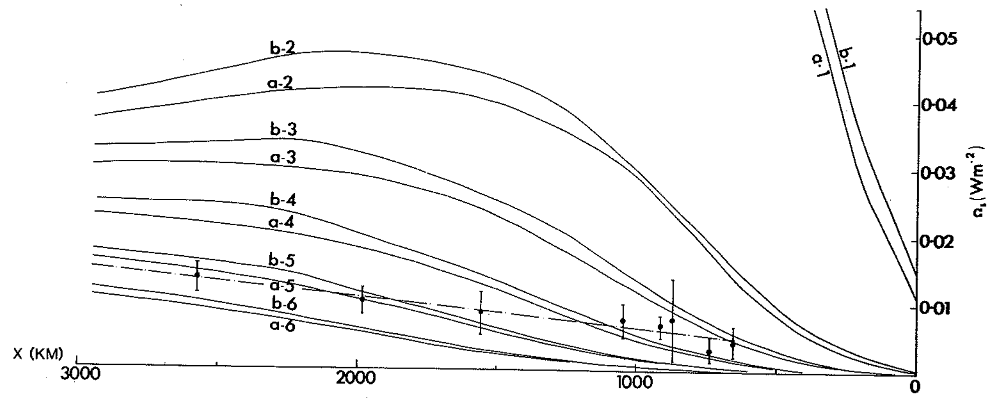

Fig. 7. Comparison of observed anomalous surface heat flow (solid circles [from *Detrick et al.*, 1981]) with the predicted results for the same parameters as used in Figure 5. The dash-dot line is the linear regression line obtained from the observation points.

away from the origin of the swell; there is no need to invoke a significant advection of heat into the shallower parts of the lithosphere.

Discussion

The two essential differences between the above model and that given by *Sandwell* [1982] are (1) the choice of the insulating lower boundary for the anomalous temperature field and (2) the inclusion of the transient effects. Sandwell adopted the isothermal lower boundary, but as illustrated in Figure 1, this causes a flux of heat back into the mantle, a flux that can be considerable if the source lies deep in the plate. For a given surface heat flow and uplift the inferred heat source intensity, dimensions, and depth are very much a function of the choice of lower boundary condition, and the application of the isothermal model to surface observable will generally lead to larger and/or shallower estimates of the sources than if the insulated model is used. Most oceanic plates move at velocities in excess of 5 cm yr^{-1} relative to the hotspots, and the age span covered by a typical volcanic island chain, assumed to be a surface expression of the hotspot trace, usually exceeds 10^7 years. Sandwell's assumption of steady state temperature field is therefore justified for most of the oceanic hotspots, provided that the plate velocity and direction as well as the hotspot intensity remained relatively constant during this interval.

The surface fields predicted by the models appear to be relatively insensitive to the thermal structure of the hotspot, partly a consequence of the "low-pass" filtering effect of the lithosphere and partly due to the trade-offs that are possible between the various parameters required in the calculations. One potential test of the conduction model is the plausibility of the required hotspot radius and intensity as deduced from such models. For example, in the case of the Hawaiian swell the available observations, particularly those defining the dimensions of the swell normal to the axis, can be matched by the conduction model if the heat source at the base of the lithosphere has a radius of about 600–700 km and an intensity of 0.5–0.6 W m^{-2}. From the results summarized in Figures 5 and 6, the thickness of the thermal lithosphere beneath Hawaii is estimated as about 40–60 km, although the agreement between the model and observations is not wholly satisfactory near the origin. The heat flow observations imply a thicker (70–90 km) thermal lithosphere (Figure 7).

These estimates assume that there is no dynamic support of the swell. One may expect that such effects will be most significant around the hotspot area such that a conductive cooling model that matches the observations at the downstream end would fail to match the observations in the hotspot area. Possibly the disparity between the above two estimates of the thickness of the lithosphere is in fact indicative of dual processes shaping the swell near the origin, but the uncertainties in filtering and smoothing the presently available data are too large to assess realistically the relative contributions of the static and dynamic effects.

Appendix 1: Solution of the Boundary Value Problem for a Temperature Disturbance

The temperature distribution $v(\mathbf{x})$ within the lithosphere can be expressed as the sum of two functions, $v = \phi + \psi$, where ψ is an unknown function and ϕ is the impulse response Green's function for infinite space. The latter is given by *Carslaw and Jaeger* [1959, p. 256] as

$$\phi = \frac{1}{8\rho c(\pi\kappa t)^{3/2}} \exp\left\{-\frac{|\mathbf{x}-\mathbf{x}'|}{4\kappa t}\right\} \tag{A1}$$

where ρ is the density of the lithosphere and c is the specific heat. Also, $\kappa = K/\rho c$, and $\mathbf{x}$ and $\mathbf{x}'$ are the position vectors of the point at which v is evaluated and of the source location, respectively. The source is applied at time $t = 0$. The function ϕ satisfies the governing equation (1) but not the boundary conditions (3). Then, since $v = \phi + \psi$, the unknown function ψ has to be the solution of the following boundary value problem:

$$\nabla^2\psi = \frac{1}{\kappa}\,\partial_t\psi \tag{A2}$$

$$\begin{aligned} -[k_1\partial_z(\phi+\psi) + h_1(\phi+\psi)]\big|_{z=0} &= 0 \\ [k_2\partial_z(\phi+\psi) + h_2(\phi+\psi)]\big|_{z=d} &= 0 \end{aligned} \tag{A3}$$

The solution of (A2) with (A3) is found by applying first the Hankel transform with the transformed quantities being denoted by tilde and the transform parameter by m. Next, the Laplace transform is applied with an asterisk for the transformed quantities and s the transform parameter. The complementary solution in the Hankel and Laplace transform domain $\tilde{\phi}^*$ is

$$\tilde{\phi}^* = \frac{1}{4\pi K\eta} \exp[-\eta|z-z'|] \tag{A4}$$

with

$$\eta = (m^2 + s/\kappa)^{1/2}$$

The double transform of (A2) is

$$\partial_{zz}\tilde{\psi}^* - \eta^2\tilde{\psi}^* = 0$$

the solution of which is

$$\tilde{\psi}^* = \frac{1}{4\pi K\eta}(A \cosh \eta z + B \sinh \eta z)$$

where A and B are unknown coefficients. The solution in the transform domain follows as

$$\tilde{v}^* = \frac{1}{4\pi K\eta}\{\exp[-\eta|z - z'|] + A \cosh \eta z + B \sinh \eta z\} \quad \text{(A5)}$$

Substituting (A5) into the transformed boundary conditions (A3) determines the constants A and B. Substituting these results back into (A5) yields

$$\tilde{v}^* = \frac{1}{2\pi K\eta}\frac{(h_1 \sinh \eta z' + k_1\eta \cosh \eta z')[\eta k_2 \cosh \eta(d - z) + h_2 \sinh \eta(d - z)]}{(k_1h_2 + k_2h_1)\eta \cosh \eta d + (k_1k_2\eta^2 + h_1h_2) \sinh \eta d} \quad \text{(A6)}$$

Taking first the inverse Laplace transform of (A6), using the method of contour integration and the residue theorem and next the inverse Hankel transform, yields the solution in the time and space domain as

$$v = \frac{1}{2\pi Kt}\exp\left(-\frac{r^2}{4\kappa t}\right)\sum_{n=1}^{\infty} B_nZ_n(z)Z_n(z')\exp(-v_n^2\kappa t) \quad \text{(A7}a\text{)}$$

The v_n are the positive roots of the following algebraic equation

$$\tan v_nd = \frac{(h_1k_2 + k_1h_2)v_n}{k_1k_2v_n^2 - h_1h_2} \qquad n = 1, 2, \cdots \quad \text{(A7}b\text{)}$$

and

$$B_n = \frac{(h_2^2 + k_2^2v_n^2)d^{-1}}{(h_1^2 + k_1^2v_n^2)[d(h_2^2 + k_2^2v_n^2) + k_2h_2] + k_1h_1(h_2^2 + k_2^2v_n^2)} \quad \text{(A7}c\text{)}$$

$$Z_n(z) = d(k_1v_n \cos v_nz + h_1 \sin v_nz) \quad \text{(A7}d\text{)}$$

The solution for the conductive heating of a moving lithosphere by a stationary heat source active for a finite time period is obtained by convolving the solution (A7) in time. Thus, if the lithosphere is moving in the x direction with a constant velocity u and if the point source heat generation in unit time is Q, the Green's function for temperature is

$$G_v(x, y, z, t) \equiv v(x, y, z, t) = \frac{Q}{2\pi Kd}\sum_{n=1}^{\infty} B_nZ_n(z)Z_n(z')I_n(x, y, t) \quad \text{(A8)}$$

where

$$I_n = \exp\left(\frac{ux}{2\kappa}\right)\int_{\zeta=0}^{t}\frac{d\zeta}{t-\zeta}\exp\left\{-\frac{r^2}{4\kappa(t-\zeta)} - \left(\frac{u^2}{4\kappa} + v_n^2\kappa\right)(t-\zeta)\right\}$$

The integral I_n can be evaluated numerically using quadrature methods. The steady state solution v^∞ follows from (A8) by taking the limit $t \to \infty$. The result is

$$v^\infty(x, y, z) = \frac{Q}{\pi Kd}\exp\left(\frac{ux}{2\kappa}\right) \cdot \sum_{n=1}^{\infty} B_nZ_n(z)K_0\left[\frac{r}{\kappa}\left(\frac{u^2}{4} + v_n^2\kappa^2\right)^{1/2}\right] \quad \text{(A9)}$$

where K_0 is the zero-order modified Bessel function of the second kind. The coordinates in these last two equations are with reference to a system fixed relative to the heat source. If desired, v^∞ can also be expressed in a moving coordinate system, attached to the lithosphere. In this case, explicit convective terms must be added [e.g., *Carslaw and Jaeger*, 1959, p. 13].

APPENDIX 2

It is possible to obtain analytical expressions for the convolution (13) for some simple distributions of heat source intensity $q(x'y')$. For example, for uniform heat over a circular area with radius A the temperature field is

$$v = \frac{2q}{Kd^2}\exp\left(\frac{ux}{2\kappa}\right)\sum_{n=1}^{\infty}\frac{Z_n(z)Z_n(z')}{v_n^2} \cdot \sum_{p=0}^{\infty}(-1)^p(2 - \delta_{0p})\cos p\theta f_{np}(r) \quad \text{(A10)}$$

where

$$f_{np}(r) = g_{np}(r) - B_{np}I_p(c_nr) \qquad 0 < r < A$$

$$f_{np}(r) = A_{np}K_p(c_nr) \qquad r \geq A$$

$$A_{np} = c_nAI_{p-1}(c_nA)I_p\left(\frac{uA}{2\kappa}\right) - \frac{uA}{2\kappa}I_{p-1}\left(\frac{uA}{2\kappa}\right)I_p(c_nA)$$

$$g_{np} = c_nrI_p\left(\frac{ur}{2\kappa}\right)[I_{p-1}(c_nr)K_p(c_nr) + K_{p-1}(c_nr)I_p(c_nr)]$$

$$B_{np} = c_nAK_{p-1}(c_nA)I_p\left(\frac{uA}{2\kappa}\right) + \frac{uA}{2\kappa}I_{p-1}\left(\frac{uA}{2\kappa}\right)K_p(c_nA)$$

$$\theta = \tan^{-1}\frac{y}{x}$$

$$c_n = \left(v_n^2 + \frac{u^2}{4\kappa^2}\right)^{1/2}$$

Acknowledgment. We thank Richard Coleman for processing GEOS 3 altimeter data and preparing the regional geoid.

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(Received September 13, 1984;
revised November 26, 1984;
accepted November 27, 1984.)